The inclusive Higgs boson cross-section in gluon-gluon fusion in soft-virtual approximation at fourth order in QCD

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Abstract

We present precise results for the inclusive Higgs boson cross-section in gluon-gluon fusion at the LHC considering state-of-the-art fourth-order results in perturbative QCD arising from the dominant soft and virtual gluon emissions. Utilizing four-loop QCD results for the gluon-form factor, the splitting function and related anomalous dimensions, we study the effects of threshold enhanced soft gluon emissions and estimate their impact on the total cross-section at the fourth order. Our study highlights the role of these higher-order contributions in improving the perturbative convergence and in significantly reducing the renormalization and factorization scale uncertainties. The results provide strong evidence for the perturbative stability and reliability of Higgs boson cross-section predictions at the LHC, thereby reinforcing the robustness of theoretical inputs in precision Higgs phenomenology. We also provide cross-section predictions using a large set of available parton distribution functions and show that, together with the value of the strong coupling $\alpha_s(m_Z)$, they cause the largest residual uncertainty for the Higgs boson cross-section in gluon-gluon fusion.

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1 Introduction

The discovery of the Higgs boson at the Large Hadron Collider (LHC) has confirmed a central pillar of the Standard Model and simultaneously opened new avenues for exploring physics beyond it. Understanding the properties of the Higgs boson is therefore essential in the search for potential signs of new physics in both current and future LHC programs. Precise theoretical predictions, combined with high-precision experimental measurements, are key to enhancing our sensitivity to possible new phenomena that may be coupled to the Higgs sector.

At the LHC, the Higgs boson is dominantly produced through the gluon-gluon fusion and receives substantial perturbative corrections in quantum chromodynamics (QCD) [1–9]. Indeed, the perturbative series for the Higgs boson production cross-section is known to converge slowly at lower orders. For instance, the next-to-leading order (NLO) QCD correction nearly doubles the leading order (LO) result. The next-to-next-to-leading order (NNLO) adds an additional $\sim 25\%$ relative to NLO, while the next-to-next-to-next-to-leading order (N3LO) contributes a further $\sim 3.5\%$ on top of the NNLO result. At N3LO, the residual theoretical uncertainty due to variations of the renormalization and factorization scales μ_R and μ_F – estimated using their standard seven-point variation – is approximately 4%. To further improve the precision of the Higgs boson cross-section, it is crucial to incorporate fourth-order QCD corrections. These higher-order contributions enhance the perturbative convergence and help reduce the sensitivity to unphysical scale choices, thereby improving the robustness of theoretical predictions.

At the LHC, the Higgs cross-section receives dominant contribution from soft gluon emissions, particularly near threshold where $z = m_H^2/\hat{s} \to 1$, with m_H being the Higgs mass and $\hat{s}$ being the partonic centre of mass energy. The effects of these soft gluons are manifested in terms of well-known large threshold logarithms in $1-z$ which in Mellin-N space, take the form $\ln^k N$ with $1 \geq k \geq 2n$ at order n. The Mellin-space formulation not only provides a remarkably accurate approximation to the exact fixed-order cross-section at N3LO [10, 11] but also facilitates the resummation of these large logarithms via threshold exponentiation techniques. This motivates the construction of an approximate fourth-order cross-section by incorporating the dominant threshold logarithms in Mellin space.

In this report, we predict the total Higgs boson cross-section in gluon-gluon fusion at fourth order in QCD using available four-loop QCD results for the gluon form factor, the splitting functions and related anomalous dimensions. After introducing the basic notation and formulae in section 2, we present a comprehensive study of Higgs boson cross-section and its uncertainties, in section 3 relevant for 13.6 TeV LHC, also using different sets of parton distribution functions (PDFs), accurate to NNLO and N3LO.

2 HEFT cross-section

To compute the fourth order QCD corrections, we use the Higgs effective theory (HEFT) and the heavy top-quark mass limit where the Higgs boson couples to gluons via effective interaction given by,

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4v} C(\mu_R^2) H G^a_{\mu\nu} G_a^{\mu\nu} , \tag{1}$$

with v being the Higgs vacuum expectation value, H the Higgs field, and $G^a_{\mu\nu}$ the gluon field strength tensor. The inclusive Higgs boson production cross-section in the gluon-gluon fusion channel in the heavy top-quark mass limit takes the form,

$$\begin{aligned}\sigma(S, m_H^2) = \tau \sum_{a,b} \int_0^1 \frac{dx_1}{x_1} \frac{dx_2}{x_2} f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \\ \times \int_0^1 dz\, \delta\left(z - \frac{\tau}{x_1 x_2}\right) \tilde{\sigma}_0^H c_{ab}^H \left(z, \alpha_s(\mu_R^2), m_H^2/\mu_R^2, m_H^2/\mu_F^2\right) ,\end{aligned} \tag{2}$$

where the Born factor in the large $m_t \to \infty$ limit reads

$$\tilde{\sigma}_0^H = \frac{\pi C(\mu_R^2)^2}{8 n_A v^2} , \quad \text{with} \quad C(\mu_R^2) = -\frac{\alpha_s(\mu_R^2)}{3\pi} \left\{1 + 11 \frac{\alpha_s(\mu_R^2)}{4\pi} + \ldots\right\} . \tag{3}$$

Here $\tau = m_H^2/S$, and $f_{a/h}(x, \mu_F^2)$ are the PDFs in the proton. The coefficient function c_{ab}^H can be expanded in strong coupling α_s with the LO term taking the form $c_{ab}^{H,(0)} = \delta_{ag}\, \delta_{bg}\, \delta(1-z)$.

The computation of the coefficient function at fourth order in the threshold limit requires several perturbative ingredients. First, the contribution from the gluon form factor, which encodes purely virtual gluon effects, is known [12, 13]. Second, the fourth-order gluon splitting functions, which describe collinear gluon emissions, are needed. These are known to a sufficient degree through the computation of several Mellin moments [14], allowing us to reliably extract the coefficient of the $1/\epsilon$ pole in the soft-virtual approximation. Furthermore, employing techniques from threshold resummation, we are able to estimate the effects of process-dependent coefficients that appear at this order.[1] In the next section, we compute the cross-section with the contribution of soft and virtual gluons at fourth order.

3 Results

It is well known that the validity of HEFT predictions can be significantly improved by rescaling them with the exact LO result computed using the full theory, which includes the finite top-quark mass and $n_f = 5$ massless quark flavors. Accordingly, to enhance the reliability of the QCD predictions incorporating fourth-order soft gluon corrections, we rescale the HEFT results in Eq. (2) by the exact LO cross-section. We present our numerical estimates for proton-proton collisions at $\sqrt{S} = 13.6$ TeV at the LHC. The Higgs boson mass is taken to be $m_H = 125$ GeV, and the on-shell top-quark mass is set to $m_t = 172.5$ GeV. Results up to N3LO are obtained using `iHixs-2` [18], whereas N4LOsv results are computed with an in-house code.

For our central scale choices, we use $\mu_R^c = \mu_F^c = m_H/2$, which is known to yield improved perturbative convergence by reducing the size of higher-order corrections. Theoretical uncertainties from scale variations are estimated using the standard seven-point method: we vary the renormalization scale μ_R and the factorization scale μ_F independently within the range $[m_H/4, m_H]$, subject to the constraint

$$\frac{1}{2} \leq \frac{\mu_R/\mu_R^c}{\mu_F/\mu_F^c} \leq 2\,, \tag{4}$$

which leads to the following seven scale combinations (in units of the central scale): $(\mu_R, \mu_F) = \{(1/2, 1/2), (1/2, 1), (1, 1/2), (1, 1), (1, 2), (2, 1), (2, 2)\} \times (\mu_R^c, \mu_F^c)$. The final scale uncertainty is then obtained by taking the envelope formed by the maximum and minimum deviations from the central prediction.

The theoretical prediction for the inclusive Higgs boson production cross-section at the LHC, as given in Eq. (2), involves the choice of a particular PDF set. PDFs, being non-perturbative and determined through fits to a global set of data, introduce an additional source of theoretical uncertainty, which must be quantified. Different PDF collaborations adopt varying methodologies and assumptions in determining these functions, leading to differences in the predicted cross-sections. To assess the impact of PDF uncertainties on our results, we consider several modern NNLO PDF sets accessed via the LHAPDF interface [19]: `ABMP16_5_nnlo` [20], `ABMPtt_5_nnlo` [21], `CT18NNLO` [22], `MSHT20nnlo_as118` [23], `NNPDF40_nnlo_as_01180` [24], `PDF4LHC21_40` [25]; and approximate N3LO PDF sets: `MSHT20an3lo_as118` [26], `NNPDF40_an3lo_as_01180` [27].

Summary of the parameters:

$$\begin{aligned} \sqrt{S} \equiv E_{\mathrm{CM}} &= 13.6 \text{ TeV}\,, & m_H &= 125 \text{ GeV}\,, & m_t &= 172.5 \text{ GeV (on-shell scheme)}\,, \\ \mu_R^c &= m_H/2\,, & \mu_F^c &= m_H/2\,, & n_f &= 5\,. \end{aligned} \tag{5}$$

[1] For a detailed discussion of the fourth-order ingredients and their computation, see [15–17].

Error estimation: The perturbative correction at order k is quantified by $\Delta(\text{N}k\text{LO})$, defined as the relative change in the cross-section at order k compared to the previous order $(k-1)$.

$$\Delta(\text{N}k\text{LO}) = \pm \left| \frac{\sigma^{\text{N}k\text{LO}}}{\sigma^{\text{N}(k-1)\text{LO}}} - 1 \right| . \tag{6}$$

This quantity serves to illustrate the perturbative convergence and to put the quantitative error estimation into perspective. In addition to uncertainties arising from scale variations and intrinsic PDF errors, we also account for several other sources of theoretical uncertainty. For the k—th perturbative order, we define:

$$\begin{aligned}
\delta^{(k)}(\text{Scale}) &= \left\{ \max\left(\sigma^{\text{N}k\text{LO}}(\mu_R,\mu_F) - \sigma^{\text{N}k\text{LO}}(\mu_R^c,\mu_F^c)\right), \min\left(\sigma^{\text{N}k\text{LO}}(\mu_R,\mu_F) - \sigma^{\text{N}k\text{LO}}(\mu_R^c,\mu_F^c)\right) \right\}, \\
\delta^{(k)}(\alpha_s) &= \pm \frac{1}{2} \frac{\left|\sigma^{\text{N}k\text{LO}}(\alpha_s^+(m_Z)) - \sigma^{\text{N}k\text{LO}}(\alpha_s^-(m_Z))\right|}{\sigma^{\text{N}k\text{LO}}(\alpha_s^c(m_Z))}, \\
\delta^{(k)}(\text{PDF}+\alpha_s) &= \sqrt{\delta^{(k)}(\text{PDF})^2 + \delta^{(k)}(\alpha_s)^2}, \\
\delta^{(k)}(\text{PDF-TH}) &= \pm \frac{1}{2} \frac{\left|\sigma^{\text{N}k\text{LO}}(\text{N3LO PDF}) - \sigma^{\text{N}k\text{LO}}(\text{NNLO PDF})\right|}{\sigma^{\text{N}k\text{LO}}(\text{N3LO PDF})},
\end{aligned} \tag{7}$$

where $\delta^{(k)}(\text{PDF})$ is the intrinsic PDF uncertainty for the perturbative order k and $\delta^{(k)}(\text{Scale})$ is the seven-point scale uncertainty described earlier. The uncertainty due to the strong coupling α_s at order k is estimated by varying $\alpha_s(m_Z)$ around its central value $\alpha_s^c(m_Z)$ by $\pm 1\sigma$. The resulting uncertainty is obtained by taking the absolute difference between the cross-sections computed with $\alpha_s^+(m_Z)$ and $\alpha_s^-(m_Z)$, and then normalizing this difference to the cross-section obtained using the central value $\alpha_s^c(m_Z)$. The combined PDF and $\alpha_s(m_Z)$ uncertainties, denoted by $\delta^{(k)}(\text{PDF}+\alpha_s)$ are obtained by adding the individual PDF and α_s uncertainties in quadrature. To estimate the theoretical uncertainty associated with the choice of PDF sets, $\delta^{(k)}(\text{PDF-TH})$ we follow the procedure described above: we compare the cross-section at N3LO using PDFs with that using NNLO PDFs, and then divide the difference by two. This estimation is only applicable to the two PDF sets for which N3LO PDFs are available, and we apply it only to the N4LOsv results, as no N4LO PDFs currently exist.

Below we consider two different scenarios corresponding to choosing $\alpha_s^c(m_Z)$:

- **Case-I: $\alpha_s^c(m_Z)$ from LHAPDF:** The PDF uncertainty is evaluated using the standard intrinsic method, employing various modern PDF sets accessed via the LHAPDF interface. It is important to note that the ABMP sets determine the strong coupling simultaneously with the PDFs, i.e. $\alpha_s(m_Z) = 0.1147 \pm 0.0008$ for `ABMP16_5_nnlo` and $\alpha_s(m_Z) = 0.1150 \pm 0.0009$ for `ABMPtt_5_nnlo`, while all other PDF sets considered here use a fixed value of $\alpha_s(m_Z) = 0.118$. In Table 1, we present the results for both N3LO and N4LOsv, including the central values of the cross-sections along with their associated scale and intrinsic PDF uncertainties. The N4LOsv correction to the cross-section is modest, amounting to a reduction of approximately −0.1% relative to N3LO. However, the associated scale uncertainty is significantly reduced by a factor of 2 at N4LOsv, while the intrinsic PDF uncertainty remains essentially unchanged. Interestingly, the use of approximate N3LO PDFs has a noticeable impact on the central values of the cross-section. For example, the NNPDF-based prediction shifts downward by about −2% and the MSHT prediction by around −5%, when compared to the corresponding values obtained using NNLO PDFs. These shifts reflect the impact of higher order corrections (beyond NNLO) on the determination of the gluon distribution, but also particular methodologies in the implementation of QCD theory predictions at N3LO within the respective PDF fits. A detailed benchmarking of different treatments of the QCD evolution of unpolarized PDFs at approximate N3LO has been performed in [28].

Table 1: Higgs cross-section at N3LO and N4LOsv for $(\mu_R^c, \mu_F^c) = (1/2, 1/2)m_H$, $\sqrt{S} = 13.6$ TeV, and $\alpha_s^c(m_Z)$ from LHAPDF (NNLO value).

	N3LO (Δ, δ in %)				N4LOsv (Δ, δ in %)			
PDF Name	Central	Δ(N3LO)	δ(Scale)	δ(PDF)	Central	Δ(N4LO)	δ(Scale)	δ(PDF)
ABMP16 [20]	48.8	3.3	$^{+0.2}_{-3.6}$	$^{+1.7}_{-1.7}$	48.7	−0.1	$^{+0.5}_{-2.1}$	$^{+1.7}_{-1.7}$
ABMPtt [21]	48.4	3.3	$^{+0.2}_{-3.6}$	$^{+1.5}_{-1.5}$	48.4	−0.1	$^{+0.5}_{-2.1}$	$^{+1.5}_{-1.5}$
CT18NNLO [22]	51.3	3.5	$^{+0.3}_{-3.9}$	$^{+2.8}_{-3.6}$	51.3	−0.1	$^{+0.5}_{-2.3}$	$^{+2.8}_{-3.6}$
MSHT20 [23]	51.4	3.5	$^{+0.3}_{-3.9}$	$^{+1.2}_{-1.2}$	51.3	−0.1	$^{+0.5}_{-2.3}$	$^{+1.2}_{-1.2}$
NNPDF40 [24]	51.7	3.5	$^{+0.3}_{-3.9}$	$^{+0.6}_{-0.6}$	51.7	−0.1	$^{+0.5}_{-2.3}$	$^{+0.6}_{-0.6}$
PDF4LHC21 [25]	51.6	3.5	$^{+0.3}_{-3.9}$	$^{+0.6}_{-0.6}$	51.5	−0.1	$^{+0.5}_{-2.3}$	$^{+0.6}_{-0.6}$
MSHT20an3lo [26]	48.7	3.5	$^{+0.3}_{-3.9}$	$^{+1.9}_{-1.7}$	48.7	−0.1	$^{+0.5}_{-2.3}$	$^{+1.9}_{-1.7}$
NNPDF40an3lo [27]	50.6	3.5	$^{+0.3}_{-3.9}$	$^{+0.6}_{-0.6}$	50.6	−0.1	$^{+0.5}_{-2.3}$	$^{+0.6}_{-0.6}$

Table 2: Approximate Higgs cross-section at N4LOsv and associated theory errors for $(\mu_R^c, \mu_F^c) = (1/2, 1/2)m_H$, $\sqrt{S} = 13.6$ TeV, $\alpha_s^c(m_Z) = 0.118$.

$k = 4$	N4LOsv (Δ, δ in %)						
PDF Name	Central	Δ(N4LO)	δ(Scale)	δ(PDF)	$\delta(\alpha_s)$	$\delta(\text{PDF} + \alpha_s)$	δ(PDF-TH)
ABMP16 [20]	53.2	−0.1	$^{+0.4}_{-2.3}$	$^{+1.0}_{-1.0}$	±4.0	$^{+4.1}_{-4.1}$	–
ABMPtt [21]	52.4	−0.1	$^{+0.4}_{-2.3}$	$^{+1.1}_{-1.1}$	±4.0	$^{+4.1}_{-4.1}$	–
CT18NNLO [22]	51.3	−0.1	$^{+0.5}_{-2.3}$	$^{+2.8}_{-3.6}$	±4.0	$^{+4.9}_{-5.4}$	–
MSHT20nnlo [23]	51.3	−0.1	$^{+0.5}_{-2.3}$	$^{+1.2}_{-1.2}$	±4.0	$^{+4.2}_{-4.2}$	–
NNPDF40nnlo [24]	51.7	−0.1	$^{+0.5}_{-2.3}$	$^{+0.6}_{-0.6}$	±4.0	$^{+4.0}_{-4.0}$	–
PDF4LHC21 [25]	51.5	−0.1	$^{+0.5}_{-2.3}$	$^{+1.7}_{-1.7}$	±4.0	$^{+4.3}_{-4.3}$	–
MSHT20an3lo [26]	48.7	−0.1	$^{+0.5}_{-2.3}$	$^{+1.9}_{-1.7}$	±4.0	$^{+4.4}_{-4.3}$	±2.7
NNPDF40an3lo [27]	50.6	−0.1	$^{+0.5}_{-2.3}$	$^{+0.6}_{-0.6}$	±4.0	$^{+4.0}_{-4.0}$	±1.1

- **Case-II: $\alpha_s^c(m_Z) = 0.118$:** The strong coupling is set to the same value of $\alpha_s^c(m_Z) = 0.118$ for all PDF sets. A 1σ variation in the strong coupling around its central value leads to $\alpha_s^+(m_Z) \equiv \alpha_s^{1\sigma+}(m_Z) = 0.1195$ and $\alpha_s^-(m_Z) \equiv \alpha_s^{1\sigma-}(m_Z) = 0.1165$. In Table 2, we present the corresponding results at N4LOsv. Noticeably, the central cross-section values obtained with the ABMP PDF sets shift almost by 10% with the choice $\alpha_s^c(m_Z) = 0.118$ compared to the values reported in Table 1. This is due the lower nominal value of $\alpha_s^c(m_Z)$ extracted by ABMP in a simultaneous fit with the PDFs. Across all the PDF sets considered, the dominant contribution to the theoretical uncertainty at this order arises from the 1σ variation in α_s, leading to an estimated uncertainty of approximately 4%.

4 Conclusion

We present predictions for the Higgs boson cross-section in gluon-gluon fusion at fourth order in perturbative QCD, incorporating the dominant contributions from soft gluon emissions in the threshold region. Soft gluons play a critical role in shaping the perturbative behavior of the cross-section, particularly near threshold. At this order, the impact of soft gluon corrections is modest, resulting in a change of approximately −0.1% relative to N3LO results. This small correction indicates excellent perturbative convergence at N4LOsv. Furthermore, the theoretical uncertainties associated with a variation of the renormalization and factorization scales in a wide range are reduced by nearly a factor of two compared to N3LO, yielding highly competitive theoretical predictions suitable for precision comparison with experimental data.

We also assess the residual theoretical uncertainties. The scale uncertainty, primarily due to soft gluon emission, is reduced to about 2% at this order, down from roughly 4% at N3LO. To evaluate PDF-related uncertainties, we examine a number of modern PDF sets, determined at NNLO accuracy. We find that the associated uncertainties for any given PDF set are generally below 2%, but the cross-section predictions obtained with all PDF sets differ among each other by up to 7%. The dominant source of uncertainty in this repect arises from the strong coupling α_s. From its variation within the 1σ range, we find an uncertainty of approximately 4%. In addition, we investigate the effect of using the approximate N3LO PDF sets `MSHT20an3lo_as118` and `NNPDF40_an3lo_as_01180`. For both sets, we observe a downward shift in the central values of the cross-sections relative to the predictions with NNLO PDFs. However, the cross-sections predictions from both sets differ among each other by 4% despite identical values of $\alpha_s(m_Z)$ used.

In summary, the Higgs boson cross section in gluon-gluon fusion is known very accurately in perturbative QCD, but the choices of the value for the strong coupling and the PDF set remain sizable sources of uncertainty, which requires further study in the future. Nevertheless, obtaining the complete N4LO correction would be highly desirable, as it would improve the accuracy of the soft-virtual approximation at this order.

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References

[1] S. Dawson, *Radiative corrections to Higgs boson production*, Nucl. Phys. B **359**, 283 (1991), doi:10.1016/0550-3213(91)90061-2.

[2] A. Djouadi, M. Spira and P. M. Zerwas, *Production of Higgs bosons in proton colliders. QCD corrections*, Phys. Lett. B **264**, 440 (1991), doi:10.1016/0370-2693(91)90375-Z.

[3] M. Spira, A. Djouadi, D. Graudenz and R. M. Zerwas, *Higgs boson production at the LHC*, Nucl. Phys. B **453**, 17 (1995), doi:10.1016/0550-3213(95)00379-7 [preprint doi:10.48550/arXiv.hep-ph/9504378].

[4] R. V. Harlander and P. Kant, *Higgs production and decay: Analytic results at next-to-leading order QCD*, J. High Energy Phys. **12**, 015 (2005), doi:10.1088/1126-6708/2005/12/015 [preprint doi:10.48550/arXiv.hep-ph/0509189].

[5] R. V. Harlander and W. B. Kilgore, *Next-to-next-to-leading order Higgs production at hadron colliders*, Phys. Rev. Lett. **88**, 201801 (2002), doi:10.1103/PhysRevLett.88.201801 [preprint doi:10.48550/arXiv.hep-ph/0201206].

[6] C. Anastasiou and K. Melnikov, *Higgs boson production at hadron colliders in NNLO QCD*, Nucl. Phys. B **646**, 220 (2002), doi:10.1016/S0550-3213(02)00837-4 [preprint doi:10.48550/arXiv.hep-ph/0207004].

[7] V. Ravindran, J. Smith and W. L. van Neerven, *NNLO corrections to the total cross section for Higgs boson production in hadron-hadron collisions*, Nucl. Phys. B **665**, 325 (2003), doi:10.1016/S0550-3213(03)00457-7 [preprint doi:10.48550/arXiv.hep-ph/0302135].

[8] C. Anastasiou, C. Duhr, F. Dulat, F. Herzog and B. Mistlberger, *Higgs boson gluon-fusion production in QCD at three loops*, Phys. Rev. Lett. **114**, 212001 (2015), doi:10.1103/PhysRevLett.114.212001 [preprint doi:10.48550/arXiv.1503.06056].

[9] B. Mistlberger, *Higgs boson production at hadron colliders at N^3LO in QCD*, J. High Energy Phys. **05**, 028 (2018), doi:10.1007/JHEP05(2018)028 [preprint doi:10.48550/arXiv.1802.00833].

[10] S. Moch and A. Vogt, *Higher-order soft corrections to lepton pair and Higgs boson production*, Phys. Lett. B **631**, 48 (2005), doi:10.1016/j.physletb.2005.09.061 [preprint doi:10.48550/arXiv.hep-ph/0508265].

[11] D. de Florian, J. Mazzitelli, S. Moch and A. Vogt, *Approximate N^3LO Higgs-boson production cross section using physical-kernel constraints*, J. High Energy Phys. **10**, 176 (2014), doi:10.1007/JHEP10(2014)176 [preprint doi:10.48550/arXiv.1408.6277].

[12] B. Agarwal, A. von Manteuffel, E. Panzer and R. M. Schabinger, *Four-loop collinear anomalous dimensions in QCD and $\mathcal{N} = 4$ super Yang-Mills*, Phys. Lett. B **820**, 136503 (2021), doi:10.1016/j.physletb.2021.136503 [preprint doi:10.48550/arXiv.2102.09725].

[13] R. N. Lee, A. von Manteuffel, R. M. Schabinger, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, *Quark and gluon form factors in four-loop QCD*, Phys. Rev. Lett. **128**, 212002 (2022), doi:10.1103/PhysRevLett.128.212002 [preprint doi:10.48550/arXiv.2202.04660].

[14] G. Falcioni, F. Herzog, S. Moch, A. Pelloni and A. Vogt, *Four-loop splitting functions in QCD - The gluon-gluon case*, Phys. Lett. B **860**, 139194 (2025), doi:10.1016/j.physletb.2024.139194 [preprint doi:10.48550/arXiv.2410.08089].

[15] G. Das, S.-O. Moch and A. Vogt, *Soft corrections to inclusive deep-inelastic scattering at four loops and beyond*, J. High Energy Phys. **03**, 116 (2020), doi:10.1007/JHEP03(2020)116 [preprint doi:10.48550/arXiv.1912.12920].

[16] G. Das, S. Moch and A. Vogt, *Approximate four-loop QCD corrections to the Higgs-boson production cross section*, Phys. Lett. B **807**, 135546 (2020), doi:10.1016/j.physletb.2020.135546 [preprint doi:10.48550/arXiv.2004.00563].

[17] B. A. Kniehl, S. Moch, V. N. Velizhanin and A. Vogt, *Flavor nonsinglet splitting functions at four loops in QCD: Fermionic contributions*, Phys. Rev. Lett. **135**, 071902 (2025), doi:10.1103/hkg5-88hr [preprint doi:10.48550/arXiv.2505.09381].

[18] F. Dulat, A. Lazopoulos and B. Mistlberger, *iHixs 2 — Inclusive Higgs cross sections*, Comput. Phys. Commun. **233**, 243 (2018), doi:10.1016/j.cpc.2018.06.025 [preprint doi:10.48550/arXiv.1802.00827].

[19] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr and G. Watt, *LHAPDF6: Parton density access in the LHC precision era*, Eur. Phys. J. C **75**, 132 (2015), doi:10.1140/epjc/s10052-015-3318-8 [preprint doi:10.48550/arXiv.1412.7420].

[20] S. Alekhin, J. Blümlein, S. Moch and R. Plačakytė, *Parton distribution functions, α_s, and heavy-quark masses for LHC run II*, Phys. Rev. D **96**, 014011 (2017), doi:10.1103/PhysRevD.96.014011 [preprint doi:10.48550/arXiv.1701.05838].

[21] S. Alekhin, M. V. Garzelli, S.-O. Moch and O. Zenaiev, *NNLO PDFs driven by top-quark data*, Eur. Phys. J. C **85**, 162 (2025), doi:10.1140/epjc/s10052-025-13832-8 [preprint doi:10.48550/arXiv.2407.00545].

[22] T.-J. Hou et al., *New CTEQ global analysis of quantum chromodynamics with high-precision data from the LHC*, Phys. Rev. D **103**, 014013 (2021), doi:10.1103/PhysRevD.103.014013 [preprint doi:10.48550/arXiv.1912.10053].

[23] S. Bailey, T. Cridge, L. A. Harland-Lang, A. D. Martin and R. S. Thorne, *Parton distributions from LHC, HERA, Tevatron and fixed target data: MSHT20 PDFs*, Eur. Phys. J. C **81**, 341 (2021), doi:10.1140/epjc/s10052-021-09057-0 [preprint doi:10.48550/arXiv.2012.04684].

[24] R. D. Ball et al., *The path to proton structure at 1% accuracy*, Eur. Phys. J. C **82**, 428 (2022), doi:10.1140/epjc/s10052-022-10328-7 [preprint doi:10.48550/arXiv.2109.02653].

[25] R. D. Ball et al., *The PDF4LHC21 combination of global PDF fits for the LHC run III*, J. Phys. G: Nucl. Part. Phys. **49**, 080501 (2022), doi:10.1088/1361-6471/ac7216 [preprint doi:10.48550/arXiv.2203.05506].

[26] J. McGowan, T. Cridge, L. A. Harland-Lang and R. S. Thorne, *Approximate N^3LO parton distribution functions with theoretical uncertainties: MSHT20aN^3LO PDFs*, Eur. Phys. J. C **83**, 185 (2023), doi:10.1140/epjc/s10052-023-11236-0 [preprint doi:10.48550/arXiv.2207.04739].

[27] NNPDF collaboration: R. D. Ball et al., *The path to N^3LO parton distributions*, Eur. Phys. J. C **84**, 659 (2024), doi:10.1140/epjc/s10052-024-12891-7 [preprint doi:10.48550/arXiv.2402.18635].

[28] A. Cooper-Sarkar, T. Cridge, F. Giuli, L. A. Harland-Lang, F. Hekhorn, J. Huston, G. Magni, S. Moch and R. S. Thorne, *A benchmarking of QCD evolution at approximate N^3LO*, (arXiv preprint) doi:10.48550/arXiv.2406.16188.