

Updated determinations of $|V_{us}|$ with tau decays using new recent estimates of radiative corrections for light-meson leptonic decay rates

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Abstract

We update the $|V_{us}|$ determinations using the HFLAV 2018 report tau branching fraction results with recent new estimates of the $\pi\ell 2$ and $K\ell 2$ radiative corrections. There are minor changes of the central values and uncertainties.



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1 Introduction

Recent measurements of $|V_{ud}|$, $|V_{us}|$ and $|V_{ub}|$ are not consistent with the unitarity condition on the first row of the CKM matrix [1,2]. Tau decay measurements are used to determine $|V_{us}|$ [3,4], supplementing the more precise determinations that are obtained using kaon decays. All these estimates also rely on lattice QCD estimates of form factors and decay constants [5,6], with the exception of the $|V_{us}|$ determinations based on the total branching fraction of the tau lepton into strange final states. Two of the $|V_{us}|$ determinations using tau measurements rely on estimates of the radiative corrections for the branching fractions $B(\tau \rightarrow \pi/K \nu)$, which are computed using also the radiative corrections for $B(\pi \rightarrow \ell \nu)$ and $B(K \rightarrow \ell \nu)$ [7]. New estimates of these radiative corrections have been computed with a novel approach using lattice QCD+QED [8]. We evaluate in the following the impact of these new estimates on the $|V_{us}|$ determinations using tau decay measurements.

2 $|V_{us}|$ from $B(\tau \rightarrow K \nu)/B(\tau \rightarrow \pi \nu)$ and $B(\tau \rightarrow K \nu)$

We use the tau branching fractions of the HFLAV 2018 report fit [3] and we compute $|V_{us}|$ using the updated external inputs provided by the Review of Particle Physics [9], by the FLAG review of lattice QCD calculations [5,6], by CODATA 2018 [10]. In updating the CODATA constants from the values used in the HFLAV 2018 report, a numerical transcription error that

slightly affected this $|V_{us}|$ determination in the HFLAV 2018 report has been corrected. Finally, we obtain $|V_{us}|$ using both the original [7] and the recently published [8] radiative corrections.

We compute $|V_{us}|$ as in the HFLAV 2018 report from the ratio of branching fractions $B(\tau^- \rightarrow K^- \nu_\tau)/B(\tau^- \rightarrow \pi^- \nu_\tau)$ and from the branching fraction $B(\tau^- \rightarrow K^- \nu_\tau)$ using the equations

$$\frac{B(\tau^- \rightarrow K^- \nu_\tau)}{B(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{f_{K^\pm}^2 |V_{us}|^2 (m_\tau^2 - m_K^2)^2}{f_{\pi^\pm}^2 |V_{ud}|^2 (m_\tau^2 - m_\pi^2)^2} \frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} (1 + \delta R_{K\mu 2/\pi\mu 2}),$$

$$B(\tau^- \rightarrow K^- \nu_\tau) = \frac{1}{16\pi} \left(\frac{G_F}{\hbar c} \right)^2 f_{K^\pm}^2 |V_{us}|^2 \frac{\tau_\tau}{\hbar} (m_\tau c)^3 \left(1 - \frac{m_K^2}{m_\tau^2} \right)^2 (1 + \delta R_{\tau/K})(1 + \delta R_{K\mu 2}),$$

respectively. We use Refs. [11–14] to get $\delta R_{\tau/K} = 0.90 \pm 0.22$, $\delta R_{\tau/\pi} = 0.16 \pm 0.14$. $|V_{ud}| = 0.97373 \pm 0.00031$ is taken from a 2020 updated determination [1]. The radiation correction terms $\delta R_{K\mu 2/\pi\mu 2} = -0.69 \pm 0.17$ and $\delta R_{K\mu 2} = 1.07 \pm 0.21$ are provided without the isospin-breaking corrections by Refs. [7, 15–18]. The same sources report also $\delta R_{\pi\mu 2} = 1.76 \pm 0.21$. The three estimates are consistent with a correlation of 0.67 between $\delta R_{\pi\mu 2}$ and $\delta R_{K\mu 2}$, which is used when computing $|V_{us}|$.

The above radiative correction factors

$$\frac{1 + \delta R_{\tau/K}}{1 + \delta R_{\tau/\pi}} (1 + \delta R_{K\mu 2/\pi\mu 2}), \quad (1 + \delta R_{\tau/K})(1 + \delta R_{K\mu 2}),$$

have recently been evaluated [19] as

$$(1 + \delta_{\tau K/\tau\pi}), \quad \text{with } \delta_{\tau K/\tau\pi} = (0.10 \pm 0.80)\%,$$

$$(1 + \delta_{\tau K}), \quad \text{with } \delta_{\tau K} = (-0.15 \pm 0.57)\%,$$

respectively, with larger and more conservative uncertainties. We do not use these last values in the following.

The above radiative correction terms are used with the lattice QCD decay constants that include isospin-breaking corrections from the FLAG 2019 lattice QCD averages with $N_f = 2+1+1$: $f_{K^\pm}/f_{\pi^\pm} = 1.1932 \pm 0.0021$ [5, 20–23] and $f_{K^\pm} = 155.7 \pm 0.3$ MeV [5, 21, 22, 24].

New estimates of the radiation correction terms inclusive of isospin-breaking corrections $\delta R'_{K\mu 2/\pi\mu 2} = -1.26 \pm 0.14$ and $\delta R'_{K\mu 2} = 0.24 \pm 0.10$ are provided by Ref. [8]. The same source reports also $\delta R'_{\pi\mu 2} = 1.53 \pm 0.19$. The three estimates are consistent with a correlation of 0.63 between $\delta R'_{\pi\mu 2}$ and $\delta R'_{K\mu 2}$, which originates from a correlation of 0.794 between $\delta R'_{\pi\mu 2}$ the non-isospin-breaking component of $\delta R'_{K\mu 2}$ [25]. These radiative correction terms are used with the isospin-symmetric lattice QCD decay constants with $N_f = 2 + 1 + 1$ provided by Ref. [8]: $f_K/f_\pi = 1.1966 \pm 0.0018$ and $f_K = 156.1 \pm 0.2$ MeV.

Table 1 reports the values of $|V_{us}|_{\tau K/\pi}$ from $B(\tau \rightarrow K \nu)/B(\tau \rightarrow \pi \nu)$ and $|V_{us}|_{\tau K}$ from $B(\tau \rightarrow K \nu)$ using both the original and the new $\pi\ell 2$ and $K\ell 2$ radiative corrections. The improvements in the precision of the radiative corrections results in minor improvements on the $|V_{us}|$ determinations because of other larger contributing uncertainties. The changes of the central values are also minor compared with the total uncertainties. Figure 1 reports the $|V_{us}|$ determinations in this document compared with $|V_{us}|$ from kaon decay measurements $K\ell 3$ [2] and $K\ell 3$ [9], and $|V_{us}|$ implied by the CKM matrix unitarity given the measured values of $|V_{ud}|$ [1] and $|V_{ub}|$ [9].

Table 1: $|V_{us}|$ determinations. $|V_{us}|_{\text{uni}}$ denotes the value of $|V_{us}|$ assuming that the CKM matrix is unitary. The determinations of $|V_{us}|_{\tau K/\pi}$ and $|V_{us}|_{\tau K}$ using the exclusive tau branching fractions are reported using both the old and the new estimates of the $\pi\ell 2$ and $K\ell 2$ radiative corrections. The signed difference with respect to $|V_{us}|_{\text{uni}}$ in standard deviations is reported for all $|V_{us}|$ determinations.

	$ V_{us} $			
$ V_{us} _{\text{uni}}$	0.2277 ± 0.0013		0.0σ	
	Cirigliano & Neufeld 2011		Di Carlo <i>et al.</i> 2019	
$ V_{us} _{\tau K/\pi}$	0.2234 ± 0.0015	-2.0σ	0.2235 ± 0.0015	-2.0σ
$ V_{us} _{\tau K}$	0.2226 ± 0.0015	-2.6σ	0.2229 ± 0.0014	-2.6σ

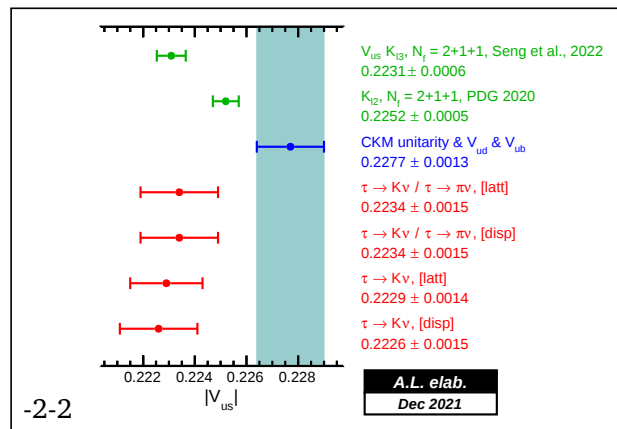


Figure 1: $|V_{us}|$ determinations using tau decay measurements with the old $\pi\ell 2$ and $K\ell 2$ radiative corrections ([disp]) and with the new ones ([latt]), compared with $|V_{us}|$ from the CKM matrix unitarity and $|V_{us}|$ from kaon measurements.

3 Conclusions

Recent new slightly more precise estimates of the $\pi\ell 2$ and $K\ell 2$ radiative corrections have been used to update the $|V_{us}|$ determination using the tau branching fractions of the HFLAV 2018 report, resulting in minor changes of the central values and uncertainties.

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