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Monte Carlos for tau lepton - Standard Model and new physics signatures

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Abstract

One of the purposes of High Energy accelerator experiments is confrontation of theory and measurements in ever new realms. Any new agreement extends theory applicability domain, any discrepancy hints to unexplained, calling for better calculations or new, deeper theory. Often one has to search for small contributions over large Standard Model background. Multidimensional signatures and complex background subtraction cuts imply that Monte Carlo techniques are indispensable. My talk included description of: KKMC Monte Carlo for $e^+e^- \to \tau^+\tau^-(n\gamma)$ (with τ decays), generation of additional lepton pairs of SM and New Physics, and contributions from anomalous magnetic and electric dipole moments.

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1 Introduction

My talk is devoted to KKMC Precision Monte Carlo for $e^+e^- \to \tau^+\tau^-(n\gamma)$, with τ decays included, and its associated projects. The main design purposes, as in the past, include: (i) precision simulation of lepton pair production, (ii) flexible set up for initialization of user defined hadronic currents used in τ decays. Recent developments in KKMC are particularly important for high energy applications: the C++ version of the software is now available and includes improvements over the FORTRAN version, such as better arrangements for beam-spread. But the most important aspect was to preserve continuity of numerical tests, an essential ingredient of work see Ref. [1]. The long road toward enhanced precision of QED part started already [2], but this road toward third order matrix elements is long and perilous. There is a need for new contributors to enter, to preserve skills and continuity of the projects, and at the same time to address new challenges.

The C++ version of KKMC is public, but is not yet adapted to lower energies (the function for photon vacuum polarization $\Pi_{\gamma\gamma}(s)$ to be obtained from dispersion relations is clearly defined, but needs to be then adjusted). There are clear manpower issues: help in installations, set-ups and of some phenomenology applications is not as robust as it was in the past. For the F77 version as installed in Belle II I take care and help, but main burden is now on somebodys else's shoulders.

Pair emission is a necessary step toward third order matrix element installation. From the algorithmic point of view, treated as radiative corrections it is incompatible with exponentiation, because the corresponding crude distribution does not feature conformal symmetry. This symmetry is essential in building a relation between crude level Monte Carlo with matrix elements of eikonal form. It is also important for matching line-shape distribution (intermediate resonances) and initial state bremsstrahlung. Alternative solutions are necessary. Real pair emission can be achieved by running of KKMC with appropriate flags for pair emission part of loop vertex correction switched on. Then a simultaneous run, of four fermion final state Monte Carlo program, like KORALW [3], is necessary. Alternatively, for final state pair emissions, Monte Carlo of the after-burn type, PHOTOS is available [4]. The program enables full phase-space coverage. Although tests with Matrix element simulations indicate that precision is sufficient for today's applications, some studies of related systematic ambiguities are included in Ref. [5]. These publications do not match the FCC precision, and further work is needed.

In parallel, some extensions with the beyond Standard Model physics are available. For τ decays see [6], and for production process, contribution to the previous TAU2021 conference [7]. The emission of dark photons (dark scalars) decaying to light lepton pairs is addressed. Extensions of KKMC Monte Carlo for four fermion final states (processes mediated by ZH boson pair) is a possible start for future, long, development path to FCC applicability.

Another pursued development direction is extension of simulation for anomalous magnetic and electric dipole moments. Recent g-2 measurements, discussed in other talks of the conference, initiate interests in anomalous dipole moments of τ lepton pair production. The question arises how to simulate their impact on differential distributions. We addressed this point with the help of algorithms enabling calculation of event weights: ratios of matrix elements for production and decay of tau lepton pairs with other one, without dipole moment effects. First it was done for lower energies $e^+e^- \to \tau^+\tau^-$ processes, as of Belle II, [8], later for higher energies, as of Z peak and above [9]. In this reference elements of calculations necessary for $pp \to \tau^+\tau^- X$ processes are discussed as well.

The process of migration from FORTRAN to C++ is advancing well. The C++ Versions of PHOTOS Monte Carlo for radiative corrections in decays [4] and KKMC [1] are already published. The TAUOLA Monte Carlo for τ decays is prepared for such transformation too, but actual implementation is not completed. Its internal structure was already prepared some time ago [10]. Further improvements are now in Belle II hands. Fits and evaluation of ambiguities of explored data are necessary. The program was prepared to run with user provided hadronic currents, which could be coded in C++ and activated with redefinition of pointers. That solution was temporarily but partly abandoned. Step back was due to requirements of Belle software organization.

My experience with PHOTOS, where migration to C++, has been completed is that the best is to translate program with young researchers' participation. They can observe that migration does not go too fast, because existing arrangements for future and for tests could be then easily lost. Participation of researchers with distinct profile is helpful, and prepares us for future takeover of the projects, independent of whoever it may be.

The software for Belle II is working well, and I am involved personally. Important contributor is now Swagato Banerjee. Installation and initialization for other platforms have recently posed problems especially for C++ version of KKMC. For example proper loading of root library was bringing difficulties, and it was reported to us, but our response was slow. That is

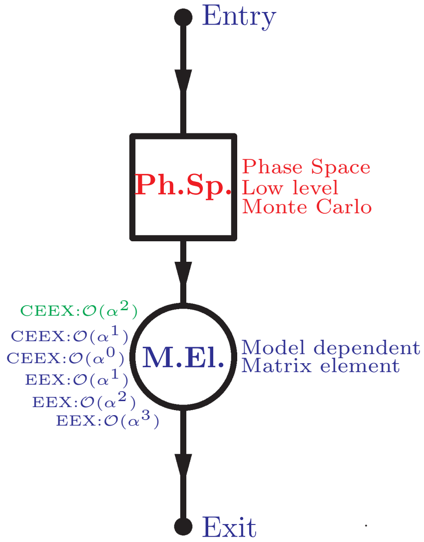

Figure 1: Structure of Matrix Element based Monte Carlo programs, in case of KKMC Plot is taken from Ref. [12].

one of the most visible aspect of the manpower issue for our community, but passing expertise to the next generation is potentially larger issue, even if related challenges are not imminent to see.

I will skip or cover most of these points briefly only, and will focus my presentation mostly on anomalous dipole moments. Let me present some details in the following sections.

2 Basics of precision Monte Carlo

Let me stress at first KKMC is based on "matrix element × full and exact phase space". Neither KKMC nor PHOTOS employ a shower-like algorithm. They are both non-Markovian. Generation starts with Poisson distribution in number of photon candidates. In case of KKMC tower of simplified/improvable structures starts from one-dimensional spectrum, then an internal generator exploiting eikonal matrix elements valid all over full multi-photon phase space is devised. It was a highly non-trivial step to achieve. Next step is exploiting bremsstrahlung part of QED matrix elements and Yennie–Frautschi–Suura exponentiation [11]. That is not all that is needed: the electroweak and strong interaction parts need to be included as well. Finally, light lepton pair emissions must be taken into account. I have omitted the complexity of presamplers necessary due to collinear configurations, which are naturally regulated by electron (or other lepton) masses. Finally, the library of Matrix Elements devised as input for event weight, sometimes called "model weight", represents an independent module see Fig. 1. The term "model weight" may be misunderstood. There is no room for arbitrary modeling. Amplitudes in use are the result of formal fixed order matrix elements. For details see [12].

Note that for phase space organization, in KKMC design, conformal symmetry of multi-photon phase space and eikonal parts of QED amplitudes is fundamental. That is why, algorithm is suitable for many applications, path for implementation of fixed order amplitudes is defined. At present, second order matrix elements are installed. In case of PHOTOS different solution is used for exact multi-photon phase space generation. It is restricted to final state radiations only, and solution is not explored beyond use of the first order matrix elements. On the other hand, emission of massive particles was easier to implement. For details seem [4,5] and references therein.

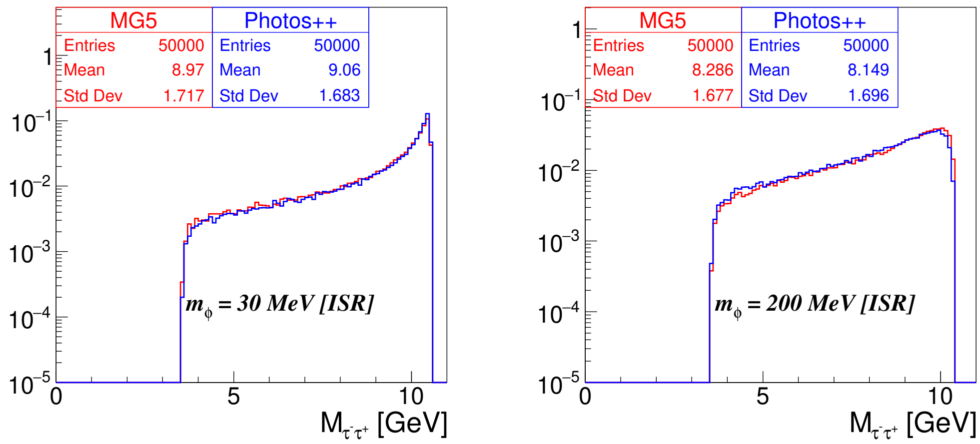

Figure 2: Belle II center of mass energies $e^-e^+ \to \tau^-\tau^+\phi_{\text{Dark Scalar}}(\to e^-e^+)$. Case of dark scalar of 30 and 200 MeV. Simulation of KKMC+PHOTOS is compared with the one based on MadGraph. Emission kernel was inspired from that comparison. At start, QED pair emission kernel was used. Spin correlations of τ-s modified by rotation of τ^- decay products. Plots taken from [13].

3 Additional pairs

For pair emissions in final states modified algorithm of PHOTOS Monte Carlo can be used. In this case phase space is treated exactly, and no approximations are used, independently of whether the pair emission is based on QED calculated matrix element or of New Physics model. As effects of pair emissions are not large, matrix elements are simplified (in improvable way though)! Tests/comparisons with matrix element based calculations are essential part of the work. As an example Fig. 2 is provided.

4 Anomalous dipole moments

One of the important aspects of Monte Carlo simulations is the possibility to imprint into precision event samples generated from Standard Model predictions, effects of New Physics. Such effects, which are expected to be small or non-existent were introduced into event re-weighting algorithms of TauSpinner [14] for embedding of anomalous dipole moment into $pp \to \tau^+\tau^-X$ simulated samples. Another algorithm for weight imprinting simultaneously with run of KKMC Monte Carlo for $e^+e^- \to l^+l^-(n\gamma)$ at low energies and also for FCC center of mass energies is also available. See Ref. [9].

For the simulation of New Physics effects, simplified kinematic for implementation is usually sufficient. There can be two weights calculated, for the cross-section:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big) / \Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big),$$

and for the spin effects related to τ lepton decays. The spin weight depends on production and on the decay of τ leptons as well:

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big) / \Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big).$$

The R_{ij} depend on the kinematic of τ-pair production, $h^i_\pm$ on $\tau^\pm$ decays. Spin quantization frames orientation needs care. It must be the same for production and decay. We use KKMC routines to transfer $h^i_\pm$ to lab frame and another routines to transfer back to $\tau^\pm$ rest frame but oriented as in New Physics calculation. In this way reference frames are easier to control and impact of photons on phase space parametrization is not of great complication. Note that the solution, if weights are calculated during KKMC run, is valid for all τ decays, while in case of TauSpinner, only for main τ decay channels spin effects are included.

In KKMC refined solution is used for compatibility with Kleiss-Stirling spinor techniques. See Ref. [15] for details. The idea was to relate τ leptons quantization frames used in production and decay through consecutive transformation from τ rest frame to lab frame and back to τ rest frame. That sounds complicated for simple rotation representation, but is safe and independent of number of bremsstrahlung photons. The $\tau^\pm$ decay products and its $h^i_{\tau^\pm}$ vector is transformed to the laboratory frame. Then $h^i_{\tau^\pm}$ is transformed back to τ lepton rest-frame, but this time the axes are oriented as chosen in Kleiss-Stirling spinor techniques.

We have explored partly this solution for KKMC anomalous moment event re-weighting. Use of host program frames is convenient but not essential. It helps to improve precision, there is no need to worry about bremsstrahlung impact etc. Use of internal program variables simplifies the tasks too. On the other hand, this prevents re-use of events for several distinct models, chosen after event samples are already stored in the data files.

The main achievements of the last year include: for KKMC usage in FCC, extension of re-weighting algorithm to FCC center of mass energies, and electroweak corrections are then included, and for TauSpinner usage in LHC, $\gamma\gamma$ parton level processes added, and explicit spin correlation matrix R_{ij} prepared for quark initialized processes as well. See Ref. [9]. With the help of TauSpinner, semi-realistic observables, see e.g. Fig. 3, have been studied. Later, it is straightforward to move to more realistic ones.

5 Conclusions and outlook

I have presented some new developments and and new applications for Monte Carlo programs family arranged around KKMC. Let me finally point to some possible, near future, activities. The big issue important for systematic errors are ambiguities of $\alpha_{QED}(s)$, the function of electromagnetic coupling. For the Monte Carlos, its values for s starting from 0 up to beyond m_Z^2 are needed. Also, the g-2 measurements revived interest in possible signatures with τ leptons: for real productions or through loops effects. I have presented some work relevant for that. But what about related systematic ambiguities?

Can modified KKMC be helpful in such studies? For $\alpha_{QED}(s)$ dispersion relations and measurements of $e^+e^- \to \pi^+\pi^-\pi^0(\pi^+\pi^-)$ at $\sqrt{s} \sim 1$ GeV^2 are necessary. This energy range can be achieved with Belle II data and events with radiative return. The invariant mass of the hadronic final state can be small if it is accompanied by hard initial state bremsstrahlung photon. The question arise if KKMC can be adapted and become helpful.

The necessary steps are as follows. Compare program predictions for $e^+e^- \to \mu^+\mu^-\gamma n(\gamma)$ at (C)EEX1 and (C)EEX2 levels.Replace $\mu^\pm$ with $\pi^\pm$ in KKMC generation with QED initial state bremsstrahlung only. Use ratio of $e^+e^- \to \pi^+\pi^-\pi^0$ to $e^+e^- \to \mu^+\mu^-$ matrix elements at interpolated phase space points of $2 \to 2$ kinematic, to calculate event weight. Imprint into final state π^0 using modified PHOTOS Monte Carlo, and make tests with alternative simulations of $e^+e^- \to \pi^+\pi^-\pi^0$ process (no bremsstrahlung photons). Reproduce old tests with PHOKARA [16, 17], see. e.g. https://indico.ph.liv.ac.uk/event/1297/contributions/7323/ . Unfortunately authors of that works are not any more available to help.

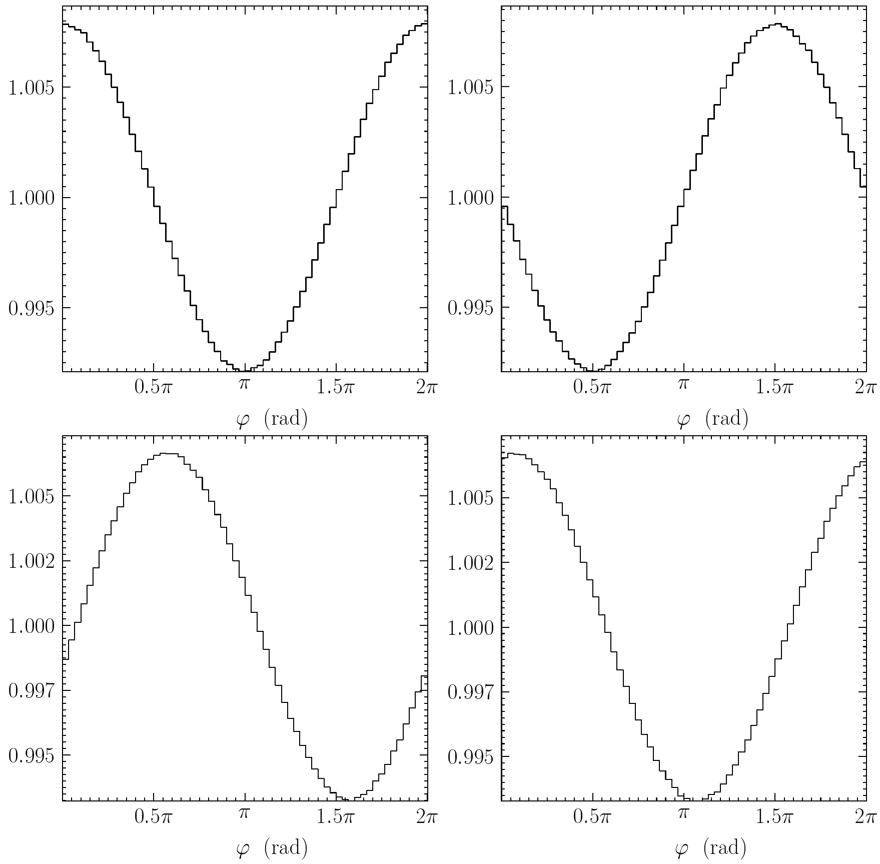

Figure 3: Ratio of number of events with and without weak dipole moments, in function of acoplanarity φ at $\sqrt{s} = M_Z$. The selected events of scattering angles $\cos(\theta) < 0$ are taken. The top left plot for $\mathrm{Re}(X) = 0.0004$, the top right plot for $\mathrm{Re}(Y) = 0.0004$, the bottom left for $\mathrm{Im}(X) = 0.0004$, and the bottom right for $\mathrm{Im}(Y) = 0.0004$ are taken. For the imaginary form-factors, additional constraint $E_{\pi^+} > E_{\bar{\nu}_\tau}$ is taken in the τ^+ side. The form-factors $A(M_Z^2) = B(M_Z^2) = 0$ are set. The decays $\tau^- \to \pi^-\pi^0\nu$ and $\tau^+ \to \pi^+\nu$ are taken. Plots taken from Ref. [9].

I have not presented further details on arrangements for τ decays and for software, except what was explained in the introduction.

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References

[1] S. Jadach, B. F. L. Ward, Z. Was, S. A. Yost and A. Siodmok, *Multi-photon Monte Carlo event generator KKMCee for lepton and quark pair production in lepton colliders*, Comput. Phys. Commun. **283**, 108556 (2023), doi:10.1016/j.cpc.2022.108556.

[2] S. Jadach, B. F. L. Ward and Z. A. Was, *Collinearly enhanced realizations of the YFS MC approach to precision resummation theory*, Phys. Lett. B **848**, 138361 (2023), doi:10.1016/j.physletb.2023.138361.

[3] S. Jadach, W. Płaczek, M. Skrzypek, B. F. L. Ward and Z. Was, *The Monte Carlo program KoralW version 1.51 and the concurrent Monte Carlo KoralW and YFSWW3 with all background graphs and first-order corrections to W-pair production*, Comput. Phys. Commun. **140**, 475 (2001), doi:10.1016/S0010-4655(01)00296-X.

[4] N. Davidson, T. Przedzinski and Z. Was, *PHOTOS interface in C++: Technical and physics documentation*, Comput. Phys. Commun. **199**, 86 (2010), doi:10.1016/j.cpc.2015.09.013.

[5] A. Kusina and Z. Was, *Evaluation of photos Monte Carlo ambiguities in case of four fermion final states*, Eur. Phys. J. C **83**, 65 (2023), doi:10.1140/epjc/s10052-023-11175-w.

[6] S. Antropov, Sw. Banerjee, Z. Was and J. Zaremba, *TAUOLA update for decay channels with e^+e^- pairs in the final state*, Comput. Phys. Commun. **283**, 108592 (2019), doi:10.1016/j.cpc.2022.108592.

[7] Sw. Banerjee, D. Biswas, T. Przedzinski and Z. Was, *Monte Carlo event generator updates, for τ pair events at Belle II energies*, (arXiv preprint) doi:10.48550/arXiv.2111.05914.

[8] Sw. Banerjee, A. Yu. Korchin and Z. Was, *Spin correlations in τ-lepton pair production due to anomalous magnetic and electric dipole moments*, Phys. Rev. D **106**, 113010 (2022), doi:10.1103/PhysRevD.106.113010.

[9] Sw. Banerjee, A. Yu. Korchin, E. Richter-Was and Z. Was, *Electron-positron, parton-parton, and photon-photon production of τ-lepton pairs: Anomalous magnetic and electric dipole moments spin effects*, Phys. Rev. D **109**, 013002 (2024), doi:10.1103/PhysRevD.109.013002.

[10] M. Chrzaszcz, T. Przedzinski, Z. Was and J. Zaremba, *TAUOLA of tau lepton decays — framework for hadronic currens, matrix elements and anomalous decays*, Comput. Phys. Commun. **232**, 220 (2016), doi:10.1016/j.cpc.2018.05.017.

[11] D. R. Yennie, S. C. Frautschi and H. Suura, *The infrared divergence phenomena and high-energy processes*, Ann. Phys. **13**, 379 (1961), doi:10.1016/0003-4916(61)90151-8.

[12] S. Jadach, B. F. L. Ward and Z. Was, *Coherent exclusive exponentiation for precision Monte Carlo calculations*, Phys. Rev. D **63**, 113009 (2001), doi:10.1103/PhysRevD.63.113009.

[13] Z. Was, *Symmetries of spin amplitudes: Applications for factorization and Monte Carlo solutions*, Proc. Sci. **406**, 008 (2022), doi:10.22323/1.406.0008.

[14] T. Przedzinski, E. Richter-Was and Z. Was, *Documentation of TauSpinner algorithms: Program for simulating spin effects in τ-lepton production at LHC*, Eur. Phys. J. C **79**, 91 (2019), doi:10.1140/epjc/s10052-018-6527-0.

[15] S. Jadach, B. F. L. Ward and Z. Was, *Global positioning of spin GPS scheme for half-spin massive spinors*, Eur. Phys. J. C **22**, 423 (2001), doi:10.1007/s100520100818.

[16] G. Rodrigo, H. Czyz and J. H. Kuhn, *Radiative return at NLO: The PHOKHARA Monte Carlo generator*, in *37th Rencontres de Moriond on electroweak interactions and unified theories* (2002).

[17] H. Czyż, *Status of Monte Carlo generators for energy scan and radiative return*, Int. J. Mod. Phys.: Conf. Ser. **35**, 1460402 (2014), doi:10.1142/S2010194514604025.