Momentum transfer dependence
of heavy quarkonium electroproduction

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Abstract

We investigate the momentum transfer dependence of differential cross sections $d\sigma/dt$ in diffractive electroproduction of heavy quarkonia on proton targets. Model predictions for $d\sigma/dt$ within the light-front QCD dipole formalism are based on a realistic model for a proper correlation between the impact parameter $\vec{b}$ of a collision and color dipole orientation $\vec{r}$. We demonstrate a significance of $\vec{b} - \vec{r}$ correlation by comparing with a standard simplification $\vec{b} \parallel \vec{r}$, frequently used in the literature.

1 Introduction

Photo- and electroproduction of heavy quarkonia represents a unique tool allowing to study diffraction mechanism, saturation phenomena, gluon distribution functions, etc. However, for a proper analysis of a given effect associated with the corresponding electroproduction process, it is essential to know various theoretical uncertainties, such as the $Q-\bar{Q}$ interaction potential, which generates quarkonium wave functions [1,2], the $Q\bar{Q} \rightarrow V$ vertex structure in connection with an open question about a contribution of the D-wave component in quarkonium wave functions [3], or the shape of dipole cross section $\sigma_{Q\bar{Q}}$, which represents one of the main ingredients within the color dipole formalism [1,2,4,5].

Theoretical and experimental investigations of the transverse momentum transfer $\vec{q}$ dependence of differential cross sections $d\sigma/dq^2$ provide the opportunity for a more detailed study of the QCD dynamics accompanying the diffractive quarkonium production. Knowledge of $\vec{q}$-orientation leads to an identification of the reaction plane since $\vec{q}$ is related to the impact parameter $\vec{b}$ of a collision via Fourier transform. Performing model predictions, this generates

a task to include a correlation between dipole orientation $\vec{r}$ and the vector $\vec{b}$ properly. Such $\vec{b} - \vec{r}$ correlation is not implemented adequately in most models for b-dependent dipole cross sections. In the present paper we rely on the model from [6] and analyze how the accurate $\vec{b} - \vec{r}$ correlation modify the results of $d\sigma/dq^2$ by comparing with predictions based on a simplified assumption $\vec{b} \parallel \vec{r}$.

The next Sec. 2 contains a short introduction to the color dipole formalism. The explicit form of the partial $Q\bar{Q}$-proton amplitude with a proper $\vec{b} - \vec{r}$ correlation is presented in Sec. 3. The impact of such $\vec{b} - \vec{r}$ correlation on magnitudes and shape of $d\sigma/dq^2$ is analyzed in Sec. 4. Our main results are summarized in Sec. 5.

2 Color Dipole Framework

Within the light-front (LF) color dipole formalism, the amplitude for electroproduction of heavy vector mesons with the transverse momentum transfer $\vec{q}$ can be expressed in the factorized form,

$$\mathcal{A}^{\gamma^* p \to V p}(x, Q^2, \vec{q}) = \langle V | \tilde{\mathcal{A}} | \gamma^* \rangle = \int d^2 r \int_0^1 d\alpha \, \Psi_V^*(\vec{r}, \alpha) \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q}) \Psi_{\gamma^*}(\vec{r}, \alpha, Q^2), \quad (1)$$

where $\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q})$ is the amplitude for elastic scattering of the color dipole on the nucleon target, $\Psi_V(r, \alpha)$ is the LF wave function for heavy quarkonium and $\Psi_{\gamma^*}(r, \alpha, Q^2)$ is the LF distribution of the $Q\bar{Q}$ Fock component of the real ($Q^2 = 0$) or virtual ($Q^2 > 0$) photon, where Q^2 is the photon virtuality and the $Q\bar{Q}$ fluctuation (dipole) has the transverse size $\vec{r}$. The variable α is the fractional LF momentum carried by a heavy quark or antiquark from a $Q\bar{Q}$ Fock component of the photon and $x = (m_V^2 + Q^2 - t)/(W^2 + Q^2 - m_N^2) \approx (m_V^2 + Q^2 - t)/s$, where m_V and m_N is the quarkonium and nucleon mass, respectively, W is c.m. energy of the photon-nucleon system and $t = -q^2$.

Most of phenomenological studies of the partial dipole amplitude $\mathcal{A}_{Q\bar{Q}}$ are performed in the impact parameter representation, where b-dependent amplitude $\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{b})$ is related to $\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q})$ in Eq. (1) via Fourier transform,

$$\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q}) = \int d^2 b \, e^{-i\vec{b}\cdot\vec{q}} \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{b}) \quad (2)$$

with the correct reproduction of the dipole cross section at $\vec{q} = 0$

$$\sigma_{Q\bar{Q}}(r, x) = \text{Im}\mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q} = 0) = 2 \int d^2 b \, \text{Im}\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}), \quad (3)$$

where the partial dipole amplitude $\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b})$ represents the interaction of the $Q\bar{Q}$ dipole acquiring orientation $\vec{r}$ with a nucleon target at the impact parameter $\vec{b}$.

The exclusive electroproduction differential cross section on a proton target reads,

$$\frac{d\sigma^{\gamma^* p \to V p}(s, Q^2, t = -q^2)}{dt} = \frac{1}{16\pi} \left| \mathcal{A}^{\gamma^* p \to V p}(s, Q^2, \vec{q}) \right|^2, \quad (4)$$

where we adopt wave functions of quarkonia generated by the realistic Buchmuller-Tye (BT) $Q - \bar{Q}$ interaction potential and a simple "S-wave-only" $V \to Q\bar{Q}$ transition requiring to perform the Melosh spin rotation [1,2,6]. The corresponding final formulas for differential cross sections can be found in [6].

3 Partial Dipole Amplitude

Let's be $\vec{r}$ the transverse separation of a colorless heavy quark $Q\bar{Q}$ photon fluctuation (dipole) and the vector $\vec{b}$ is the impact parameter of its centre of gravity. Then the corresponding interaction of the $Q\bar{Q}$ dipole is possible due to the difference between impact parameters of Q and $\bar{Q}$ relative to the scattering centre. Thus, independently of the magnitude of $\vec{r}$, the production of any $Q\bar{Q}$ component with the same impact parameter from the target related to Q and $\bar{Q}$ is terminated (see Fig. 1). This leads to vanishing and maximal dipole interaction if $\vec{b} \perp \vec{r}$ and $\vec{b} \parallel \vec{r}$, respectively.
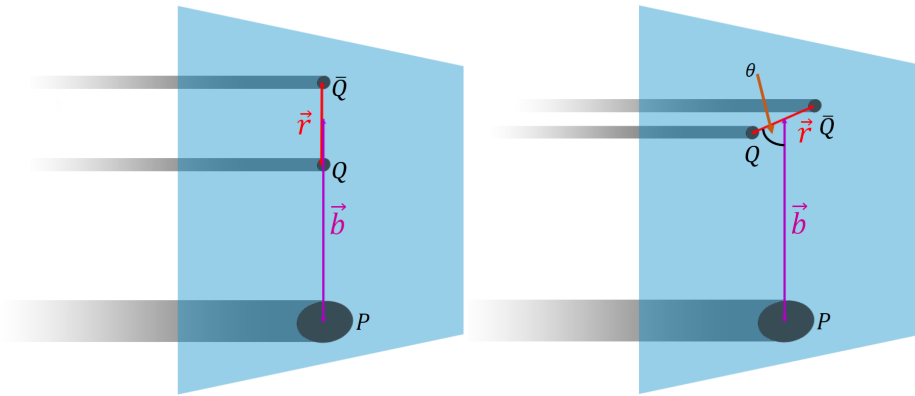
Figure 1: A cartoon demonstrating a significance of dipole orientation. Whereas the left panel illustrates the standard approximation $\vec{r} \parallel \vec{b}$ (an angle between vectors $\vec{r}$ and $\vec{b}$ is fixed at $\theta = 0^o$), the right panel represents the general case with no restrictions for an angle θ requiring so a subsequent integration over θ in calculations.

Such a correlation between the vectors $\vec{b}$ and $\vec{r}$ is incorporated in the partial elastic dipole amplitude $A_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b})$ introduced in [7] within the standard model for the dipole cross section of a conventional saturated form, $\sigma_{Q\bar{Q}}(r, x) = \sigma_0 \left(1 - \exp\left[-r^2/R_0^2(x)\right]\right)$. The corresponding partial dipole apmlitude reads [6–8],

$$\mathrm{Im}\mathcal{A}_{Q\bar{Q}}^N(\vec{r}, x, \alpha, \vec{b}) = \frac{\sigma_0}{8\pi\mathcal{B}(x)} \left\{ \exp\left[-\frac{\left[\vec{b} + \vec{r}(1-\alpha)\right]^2}{2\mathcal{B}(x)} \right] + \exp\left[-\frac{(\vec{b} - \vec{r}\alpha)^2}{2\mathcal{B}(x)} \right] \right.$$
$$\left. - 2\exp\left[-\frac{r^2}{R_0^2(x)} - \frac{\left[\vec{b} + (1/2 - \alpha)\vec{r}\right]^2}{2\mathcal{B}(x)} \right] \right\}, \qquad (5)$$

where $\mathcal{B}(x) = R_N^2 + R_0^2(x)/8$ with R_N^2 related to the constant term in the standard Regge parametrization for the energy-dependent t-slope of the differential elastic cross section. In our calculations we adopt the GBW dipole model [9, 10], where the above parameters read: $\sigma_0 = 23.03$ mb, $R_0(x) = 0.4$ fm $\times (x/x_0)^{0.144}$ with $x_0 = 3.04 \times 10^{-4}$. The form of the dipole amplitude (5) correctly reproduces at $\vec{q} = 0$ the dipole cross section according to Eq. (3).

4 Results and discussions

To quantify a significance of the correlation between vectors $\vec{b}$ and $\vec{r}$, we compare our calculations with those based on the simplified assumption $\vec{b} \parallel \vec{r}$. This is shown in Fig. 2, which demonstrates an importance of the dipole orientation in the partial amplitude $\mathcal{A}_{Q\bar{Q}}^N$, Eq. (5),

(solid lines) with respect to the $\vec{b} \parallel \vec{r}$ case (dashed lines) at c.m. energies $W = 50$ (left panel) and 200 GeV (right panel). In order to study a net effect of $\vec{b} - \vec{r}$ correlation itself, we omit in calculations the corrections for the real part of the production amplitude and the skewness effect.
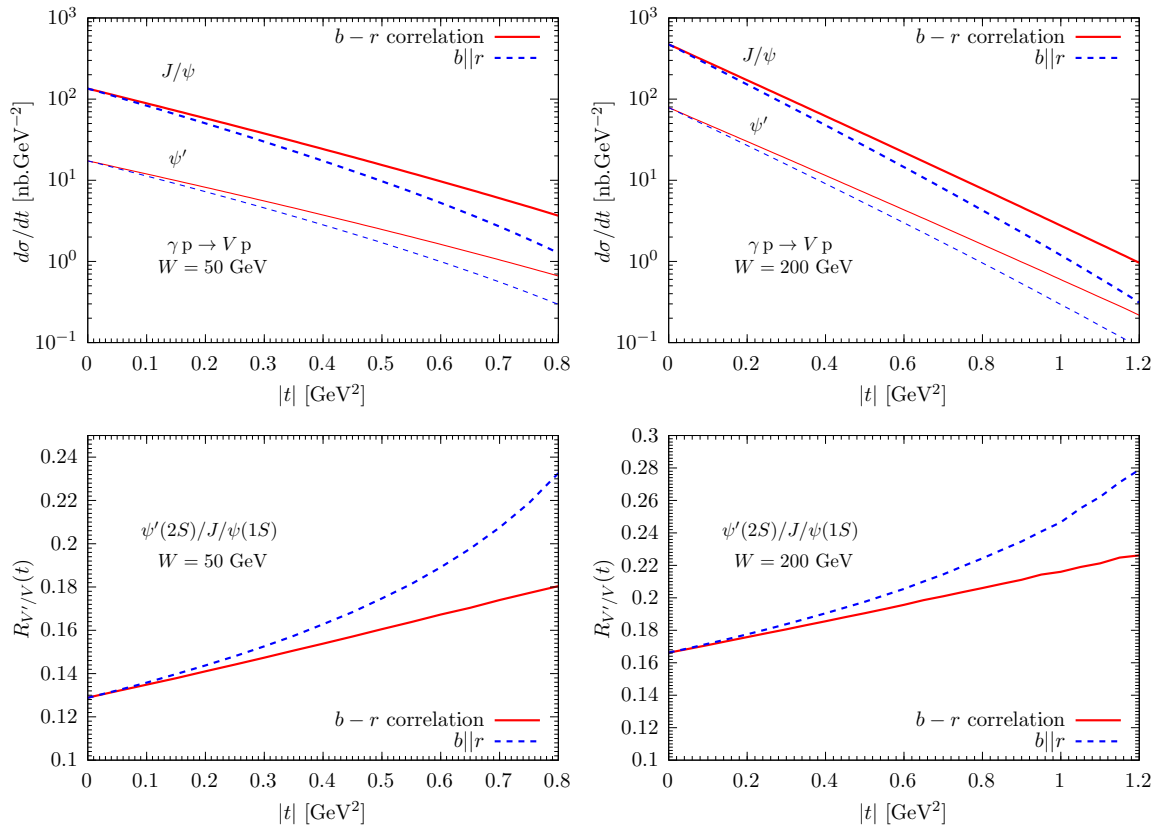
Figure 2: Manifestation of an importance of a proper $\vec{b} - \vec{r}$ correlation in the partial elastic dipole amplitude by performing calculations of differential cross sections $d\sigma^{\gamma p \to J/\psi(\psi')p}(t)/dt$ (top panels) and $\psi'(2S)$-to-$J/\psi(1S)$ ratio of differential cross sections (bottom panels) at c.m. energies $W = 50$ (left panels) and 200 GeV (right panels).

One can see from top panels of Fig. 2 that our model predictions for $d\sigma/dt$ including a realistic $\vec{b} - \vec{r}$ correlation in the partial dipole amplitude (5) differ significantly from results based on a simplified assumption when $\vec{b} \parallel \vec{r}$ (see differences between solid and dashed lines). The corresponding t-slopes of $d\sigma/dt$ are rather different, what has an indispensable impact on all predictions where authors assume that the dipole amplitude is independent of the angle between vectors $\vec{b}$ and $\vec{r}$. The significance of dipole orientation rises towards smaller photon energies corresponding to the energy range of experiments at the LHC and future electron-ion colliders. This gives a possibility to eliminate various models for b-dependent dipole amplitude from the description of diffractive quarkonium electroproduction.

The node effect in production of the $\psi'(2S)$ state can be investigated through the t-dependent $\psi'(2S)$-to-$J/\psi(1S)$ ratio $R_{\psi'/J/\psi}(t) \equiv R(t)$ of differential cross sections as is demonstrated in bottom panels of Fig. 2. One can see that it causes a rather steep rise of $R(t)$, which is gradually changed for a more flat t-behavior at higher photon energy due to a weaker node effect. Let's suppose that $\vec{b} - \vec{r}$ correlation in the dipole amplitude is not included properly. In that case, we predict a much stronger rise with t of the ratio $R(t)$ especially at the smaller photon energy (see differences between dashed and solid lines in bottom panels of Fig. 2).

This represents another way of ruling out various b-dependent models describing the partial dipole amplitude.

5 Conclusions

We studied the impact of a proper $\vec{b} - \vec{r}$ correlation in the partial dipole amplitude on magnitudes of differential cross sections $d\sigma/dt$ for diffractive electroproduction of heavy quarkonia on proton targets.

We demonstrated that, in comparison with a correct $\vec{b} - \vec{r}$ correlation, usual approximation $\vec{b} \parallel \vec{r}$ leads to a larger t-slope of $d\sigma/dt$ and causes much steeper rise with t of $\psi'(2S)$-to-$J/\psi(1S)$ ratio of differential cross sections.

The significance of dipole orientation becomes stronger towards smaller photon energies and can be tested by experiments at the LHC and future electron-ion colliders.

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