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QCD factorization for hadronic quarkonium production at high p_T

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Abstract

Heavy quarkonium production at high transverse momentum (p_T) in hadronic collisions is explored in the QCD factorization approach. We find that the leading power in the $1/p_T$ expansion is responsible for high p_T regime, while the next-to-leading power contribution is necessary for the low p_T region. We present the first numerical analysis of the scale evolution of coupled twist-2 and twist-4 fragmentation functions (FFs) for heavy quarkonium production and demonstrate that the QCD factorization approach is capable of describing the p_T spectrum of hadronic J/ψ production at the LHC.

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1 Introduction

Understanding heavy quarkonium production is a challenging and exciting research subject in the study of QCD. NRQCD factorization [1] has successfully described many features of existing data. However, at the LHC energies, significant enhanced contributions in powers of $\ln(p_T^2/m^2)$ are not fully included in fixed order NRQCD calculations, affecting the shape of calculated p_T spectrum of heavy quarkonium production.

The renormalization group improved QCD factorization approach is capable of studying such logarithmically enhanced higher-order contributions systematically [2,3]. The QCD factorization approach expands the p_T spectrum in powers of $1/p_T$ first. It factorizes both the leading power (LP) and next-to-leading power (NLP) contributions in terms of perturbatively

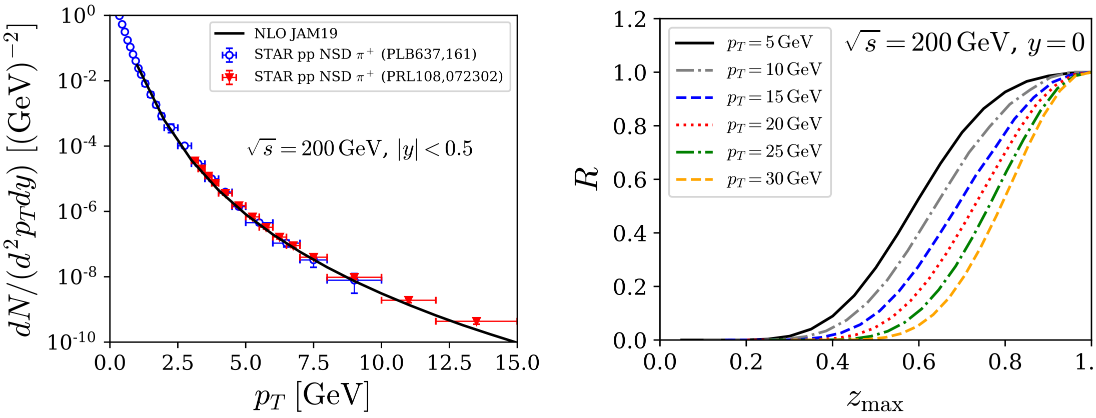

Figure 1: (Left): Differential cross section for π^+ production in $p+p$ collisions at mid rapidity at RHIC. Data are from Refs. [4, 5]. (Right): The normalized hadronic π^+ production as a function of $z_{\max}$, see text for the definition of R and $z_{\max}$.

calculable hard parts (expanded in powers of α_s) convoluted with universal parton distribution functions (PDFs) and fragmentation functions (FFs). This approach is reliable if the uncertainty from all neglected contributions in higher powers of α_s and higher powers of $1/p_T$ are controllable, and its predictive power relies on the universality of the PDFs and FFs. This paper shows that the QCD factorization approach can describe the prompt J/ψ production in hadronic collisions ($A+B \to J/\psi(p)+X$) at the LHC energies, and argue that both LP and NLP contributions are essential for describing the shape of J/ψ's p_T spectrum.

2 Quarkonium production in QCD factorization

2.1 LP contribution

Inclusive production of a single hadron of mass m_h at high p_T ($\gg m_h$) in hadronic collision can be factorized in QCD as [2],

$$E_p \frac{d\sigma_{A+B\to H(p)+X}}{d^3p}\bigg|_{\rm LP} = \sum_{f=u,d,s,c} \int_{z_{\min}}^{1} \frac{dz}{z^2} D_{f\to H}(z,\mu^2) E_c \frac{d\hat{\sigma}_{A+B\to f(p_c)+X}}{d^3p_c}\left(p_c = \frac{p}{z},\mu^2\right), \quad (1)$$

where $\hat{\sigma}_{A+B\to f(p_c)+X}$ represents the cross section to produce the fragmenting parton of flavor f and momentum p_c with all collinear sensitivities around $p_c \sim p/z$ absorbed into the twist-2 parton-to-hadron FFs, $D_{f\to H}$ with momentum fraction z and factorization scale μ, and can be further factorized into PDFs of colliding hadrons and perturbatively calculable hard parts, which are available at the next-to-leading (NLO) accuracy in α_s expansion [6]. Corrections to (1) are expected to be suppressed by the power of $1/p_T$.

The factorization formula in (1) has been successful in interpreting data on light hadron production, such as STAR-data for π^+ production in $p+p$ collisions at RHIC, as shown in Fig. 1 (Left). The theory curve was obtained by using JAM19 sets [7] for PDFs and π^+ FFs with $\mu^2 = p_T^2$. We find a nice agreement between the theoretical curve and data points for $p_T \gtrsim 1\,\mathrm{GeV}$, which indicates that $\ln(p_T^2/m^2)$-type logarithmically enhanced contributions start to dominate when $p_T/m \gtrsim 5$ (or 7) with $m \sim \Lambda_{\rm QCD}$ (or $m \sim m_\pi$) and power corrections in m/p_T are sufficiently small. We note that high p_T hadron production in $p+p$ collisions is more sensitive to the FFs at large z (in comparison with the production in e^+e^- or e^-p collisions) due to the steep falling nature of PDFs of two colliding hadrons at large momentum fraction x.

To quantify this feature, we plot $R \equiv \left[\int_{z_{\min}}^{z_{\max}} dz/z^2 D_{f\to\pi^+} d\hat{\sigma}\right] / \left[\int_{z_{\min}}^{1} dz/z^2 D_{f\to\pi^+} d\hat{\sigma}\right]$ in Fig. 1 (Right), where about 50% of the cross section results from $z = 0.7$ and above at $p_T = 15\,\text{GeV}$, while $D_{f\to\pi^+}$ is falling fast when z increases. This feature is specially relevant to heavy quarkonium production since its fragmentation functions are likely peaked in the large z region.

The factorization formalism in (1) should be applicable to J/ψ production when $D_{f\to\pi}$ is replaced by $D_{f\to J/\psi}$, so long as the power corrections in $1/p_T$ are sufficiently small and the $\ln(p_T^2/m^2)$-type contributions dominate the production cross section [2]. Since it is necessary to have a $c\bar{c}$ pair to form a J/ψ, the fragmenting parton should have a minimum virtuality, $m \gtrsim 2m_c \gg m_\pi$. If we require the similar dominance of $\ln(p_T^2/m^2)$-type contributions to the π production, we expect the formula in (1) to work for J/ψ production when $p_T \gtrsim 5\,(\text{or}\,7) \times (2m_c) \sim 15-20\,\text{GeV}$. Since producing a high p_T $c\bar{c}$ pair at the hard collision is suppressed by $1/p_T^2$ comparing the production of single fragmenting parton, the LP formalism in (1) covers only events where $c\bar{c}$ pairs emerge at distances longer than $1/\mu_0$ with $\mu_0 \sim 2m_c$ - the scale of non-perturbative input FFs, $D_{f\to J/\psi}(z,\mu_0^2)$. As shown in Sec. 3, the factorized LP contribution provides a good description of the published LHC data at high $p_T \gtrsim 60\,\text{GeV}$, but is far below the data when extrapolated to lower p_T.

2.2 NLP contribution

The NLP contributions to the inclusive production of a single hadron at high p_T can also be factorized and could be particularly important for heavy quarkonium production [3]:

$$E_p \frac{d\sigma_{A+B\to H(p)+X}}{d^3p}\bigg|_{\text{NLP}} \approx \sum_\kappa \int \frac{dz}{z^2} D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu^2) E_c \frac{d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)](p_c)+X}}{d^3p_c}\left(p_c = \frac{p}{z},\mu^2\right), \quad (2)$$

where $\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)](p_c)+X}$ represents the cross section to produce a fragmenting $Q\bar{Q}$ pair of spin-color state κ and momentum $p_c = P_Q + P_{\bar{Q}} = P'_Q + P'_{\bar{Q}}$, where P'_Q and $P'_{\bar{Q}}$ are momenta in the conjugated production amplitude, with all collinear sensitivities around p_c absorbed into the twist-4 $Q\bar{Q}(\kappa)$-to-hadron FFs, $D_{[Q\bar{Q}(\kappa)]\to H}$. For simplicity, in this paper, we approximate $P_Q = P_{\bar{Q}} = P'_Q = P'_{\bar{Q}} = p/(2z)$ [3]. Although corresponding partonic hard parts to produce a pair of heavy quarks are $1/p_T^2$ suppressed, the NLP contribution could be important since it is more likely to get the quarkonium from a fragmenting $Q\bar{Q}$-pair than a single fragmenting parton [3]. With the $1/p_T^2$ suppressed hard parts at LO, derived in Ref. [8], as shown in Sec. 3, we find that the NLP contribution provides the much needed enhancement at low p_T to improve the overall description of the LHC data from the QCD factorization approach.

2.3 Renormalization group improvement

Physically observed cross sections should not depend on the factorization approach to describe them. Renormalization group improved QCD factorization at the NLP accuracy requires the twist-2 and twist-4 FFs to satisfy the following coupled evolution equations [3],

$$\frac{\partial}{\partial \ln\mu^2} D_{f\to H}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} \sum_{f'} \int_z^1 \frac{dz'}{z'} P_{f\to f'}\left(\frac{z}{z'}\right) D_{f'\to H}(z',\mu^2)$$
$$+ \frac{\alpha_s^2(\mu)}{\mu^2} \sum_\kappa \int_z^1 \frac{dz'}{z'} P_{f\to[Q\bar{Q}(\kappa)]}\left(\frac{z}{z'}\right) D_{[Q\bar{Q}(\kappa)]\to H}\left(z',\mu^2\right), \quad (3)$$

$$\frac{\partial}{\partial \ln\mu^2} D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu^2) = \frac{\alpha_s(\mu)}{2\pi} \sum_n \int_z^1 \frac{dz'}{z'} P_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(\frac{z}{z'}\right) D_{[Q\bar{Q}(n)]\to H}(z',\mu^2), \quad (4)$$

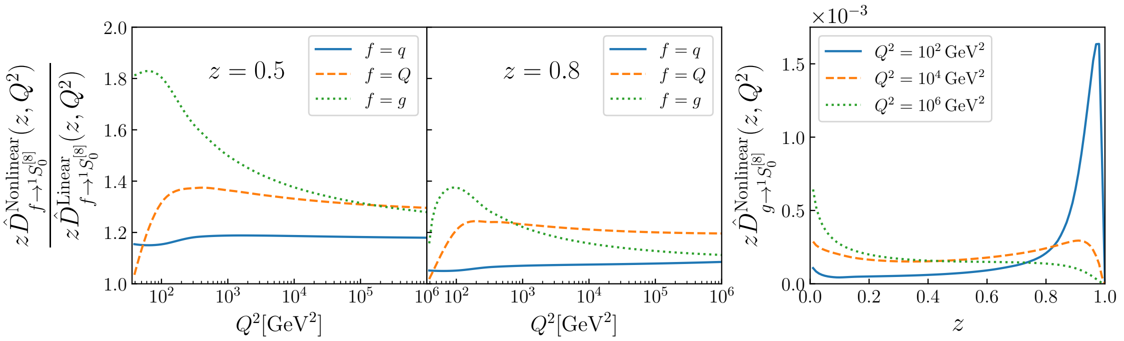

Figure 2: (Left): Ratios of the LP FFs with the nonlinear quark pair corrections to that without the nonlinear corrections for $\hat{D}_{f\to {}^1S_0^{[8]}} \equiv D_{f\to H}/\langle \mathcal{O}^H_{{}^1S_0^{[8]}}\rangle$. (Right): The $\hat{D}_{g\to {}^1S_0^{[8]}}$ LP FF with the nonlinear corrections as a function of z.

where the first line of (3) is the well-known DGLAP evolution of the twist-2 FFs as a consequence of requiring the renormalization group improved QCD factorization at the LP accuracy, and the second line of (3) represents a NLP contribution to the DGLAP evolution, which effectively resums logarithmically enhanced contributions to the cross section when the produced fragmenting parton fragments to a heavy quark pair at a scale between $[\mu_0, \mu \sim p_T]$, and the pair then fragments to the observed quarkonium H. In (3), the evolution kernels, $\alpha_s^2(\mu) P_{f\to[Q\bar{Q}(\kappa)]}(z) \equiv \gamma_{f\to[Q\bar{Q}(\kappa)]}\left(u = \frac{1}{2}, v = \frac{1}{2}, z\right)$ with $\gamma_{f\to[Q\bar{Q}(\kappa)]}$ are given in Ref. [3]. In (4), the evolution kernels,
$\frac{\alpha_s(\mu)}{2\pi} P_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}(z) \equiv \int_0^1 du \int_0^1 dv\, \Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(u, v, u' = \frac{1}{2}, v' = \frac{1}{2}, z\right)$
with $\Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}$ are given in Ref. [3].

At the NLP accuracy, the renormalization group improved and factorized cross section covers all events in which the heavy quark pair can be produced at the short-distance (2), at the input scale (1), or in-between (3). Unlike the power corrections to the cross section in (2), which go away by powers of $1/p_T^2$, the contribution to the cross section from the power correction to the evolution of twist-2 FFs in (3) remains important even at large p_T because its contribution to the cross section is built up from μ_0 to $\mu \sim p_T$ and heavy quarkonium FFs are peaked in the large z region.

3 Numerical results for J/ψ production

The predictive power of the QCD factorization approach to J/ψ production at high p_T, combining Eqs. (1)-(4), relies on our knowledge of the non-perturbative twist-2 and twist-4 FFs. With the heavy quark mass $m_c \gg \Lambda_{\mathrm{QCD}}$, as a model, we could apply NRQCD factorization to express the analytic twist-2 and twist-4 FFs at the input scale, $\mu_0 \gtrsim 2m_c$ in terms of a small set of NRQCD long-distance-matrix-elements (LDMEs) with their z-dependence calculated perturbatively in NRQCD in an expansion of α_s and heavy quark velocity v in the pair's rest frame [2, 3, 8]. Both twist-2 and twist-4 FFs at the input scale μ_0 have been derived for both LO and NLO, and expressed in terms of four LDMEs corresponding to the pair in spin-color states: ${}^3S_1^{[1]}$, ${}^1S_0^{[8]}$, ${}^3S_1^{[8]}$, ${}^3P_J^{[8]}$ with $J = 0, 1, 2$ [9, 10]. In principle, one could solve the evolution equations in (3) and (4) with the NRQCD calculated input FFs at μ_0, and use calculated hard parts and the QCD factorization formalisms in (1) and (2) to predict the J/ψ's p_T spectrum at the LHC energies.

In practice, perturbatively calculated FFs are only well-defined under the integration over

z due to their dependence on (i) $\delta(1-z)$, (ii) $f(z)\ln(1-z)$ and (iii) $f(z)/[1-z]_+$ and $f(z)[\ln(1-z)/(1-z)]_+$ with $f(z)$ a regular function and the standard "+" prescription for $[...]_+$. As functions of z, these types of contributions to input FFs as perturbative coefficients of α_s^n with $n=1,2$ could be much larger than one, for example, as $z \to 1$, making the perturbative expansion not reliable. Furthermore, gluon radiation to neutralize a fragmenting $c\bar{c}$ pair's color necessarily requires $D_{[Q\bar{Q}(\kappa)]\to J/\psi}(z) \to 0$ as $z \to 1$, while the $\delta(1-z)$ and $f(z)\ln(1-z)$ dependence from the fixed order perturbative calculations leads to an unphysical infinity as $z \to 1$. Even under the integration, if we solve the evolution equations in terms of Mellin moments, the fact that the FFs dominate at the large z requires special care for taking the inverse to get the evolved distributions as functions of z. Therefore, instead of worrying about the perturbative stability and size of higher order corrections, in this paper, we model these three types of contributions to the input FFs as $N z^{\alpha}(1-z)^{\beta}/B[1+\alpha, 1+\beta]$, where α and β are free parameters, B is the Euler Beta-function, and N is equal to the first moment of the corresponding term, which takes into account the relative size of different terms from perturbative calculations [9, 10]. If the first moments are negative, we take the absolute values to keep the same order of magnitude for the contributions. For all other contributions that vanish at $z=1$, we use corresponding analytical expressions. With different choices of (α, β), we could make the z-dependent contribution peaked at any value of z.

In our numerical calculations, we set $\alpha = 30$, $\beta = 0.5$ to have the input FFs peaked near $z=1$ for both the twist-2 and twist-4 input FFs at $\mu_0 = 4m$ with $m = m_{J/\psi}/2 \approx 1.5\,\text{GeV}$ a charm quark mass embedded in the input FFs. In Fig. 2 (Left), we demonstrate that the impact of the nonlinear quark pair correction in (3) does not disappear even at higher scales since the correction is to the evolution slope of FFs, not the FFs themselves [11]. As a result, the twist-2 FFs at large z can be enhanced by about 10–30% due to the quark pair correction even at a large probing scale. In Fig. 3, we compare our calculation with CMS data on prompt J/ψ production in the rapidity bin $|y| < 1.2$ [12, 13]. We find that the production is dominated by the ${}^1S_0^{[8]}$ channel, and we fix the LDME $\langle \mathcal{O}^{J/\psi}_{{}^1S_0^{[8]}} \rangle$ by fitting the LP contribution without the nonlinear quark pair corrections to CMS data at $p_T = 60\,\text{GeV}$ and above at $\sqrt{s} = 7, 13\,\text{TeV}$. Using CT18NLO set for PDFs [14], we obtained $\langle \mathcal{O}^{J/\psi}_{{}^1S_0^{[8]}} \rangle = 0.129 \pm 5.18 \times 10^{-3}\,\text{GeV}^3$, which is similar to the one obtained by NLO fixed order NRQCD calculations [15]. We show the ratios between CMS data and three sorts of theoretical results in Fig. 3(Left). The NLO LP contribution with the nonlinear quark pair corrections describes CMS data at high p_T, while the NLP contribution becomes more significant around $p_T = 30\,\text{GeV}$ and below. Since we used the NLP partonic cross section at LO in our calculations, we could include a K-factor to mimic NLO contributions. In Fig. 3(Right), a nice agreement between theoretical results and CMS data can be achieved with the chosen (α, β) and $K_{\text{NLP}} = 2$.

4 Conclusion

We presented the first numerical calculations for J/ψ production in hadronic collisions in the renormalization group improved QCD factorization formalism, including the NLP contribution. We demonstrated that the LP contributions dominate the high p_T region, while the NLP contributions are sizable at lower p_T and necessary for describing the LHC data within the QCD factorization approach. With only two parameters (α, β) and K_{NLP}, theoretical calculations in terms of QCD factorization are consistent with existing data while there is sufficient room to improve.

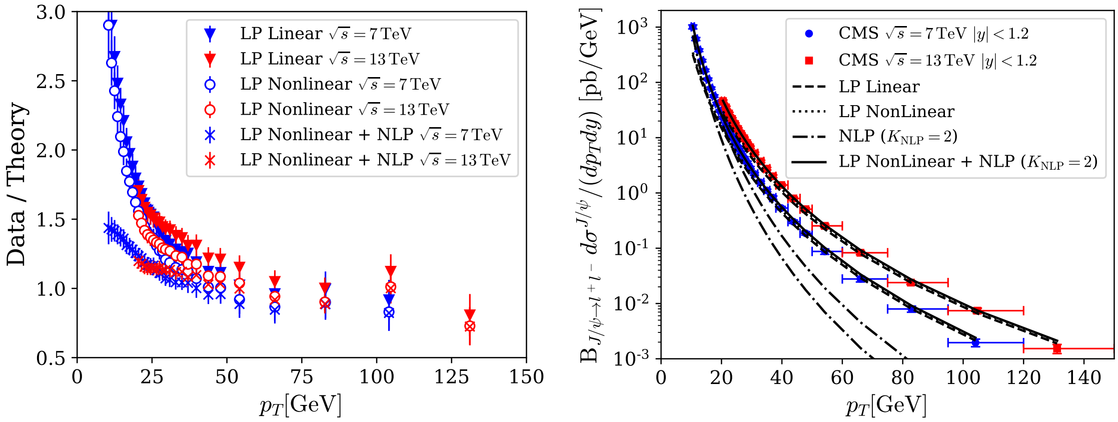

Figure 3: (Left): Ratios of CMS data to theoretical calculations for hadronic J/ψ production at the LHC. (Right): Prompt J/ψ production in $p+p$ collisions at mid rapidity at the LHC with $K_{\rm NLP} = 2$ and (α, β) given in the text.

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