# Response to 1st Report

#### December 6, 2021

We would like to acknowledge the work of the Referee in producing detailed comments. Unfortunately, it appears that some crucial aspects of the manuscript might have gone unnoticed when preparing the review. Thus, we would like comment on the points raised by the referee, hoping that this clarifies the goals and methods of the project.

# **QED** Contributions

It is not necessary to resort to NLO parton showers to embed interference. Any iterated shower may produce the final state with a known rate, which can be re-weighted to the correct rate, including the interference between QCD and QED. The shower acts as phase-space generator, which produces momentum configurations on which any squared (sum of) matrix elements (including interferences) may be evaluated. We use iterative matrix element corrections, as described in II.C, to achieve this. We refer the Referee to this crucial aspect of the manuscript – which is not mentioned in the report – to resolve the misunderstanding. Once we have also received comments from the second referee, we aim at including an additional sentence in the manuscript to again stress the importance of matrix element corrections for our approach.

#### The colour evolution, Sec. II.B

The Referee's comments from this point on seem to assume that we are aiming for amplitude-level evolution. While amplitude-level evolution might be a tool to consistently sum up  $1/N_c^2$  effects, we are rather building upon the factorization of the real-emission pattern at the cross-section level. The color-flow basis is used as a way to sample the fundamental colour indices of the produced states, which characterize the partonic state that is to be corrected through matrix-element corrections. Once we have also received comments from the second referee, we aim at including an additional sentence to stress that no amplitude-level evolution is attempted.

## Question I: What represents the partonic state?

The referee seems to assume here that we aim to perform amplitude-level evolution, and in fact that amplitude-level evolution is the only possible tool to embed subleading- $N_c$  corrections within parton showers. The vast amount of literature concluding differently (see e.g. the small selection [1, 2, 3, 4, 5]), in particular when focusing on improved real-emission descriptions in showers, puts this assessment in doubt.

To address the question in the section title: The partonic state is represented by a set of fourmomenta, flavors and fundamental colour indices for the final state partons.

# Question II: What is the real splitting operator?

The splitting kernels employed in the fixed-color parton shower are explained in detail in sec. II.B, in particular in eqs. 8-11. Further details are given in [14], and in the flowchart in the appendix. The correct color-correlator prefactor for each splitting is instated by the procedure explained on page 5. As a purely implementational method, the color correlator is calculated by performing the colour insertion

independently on auxiliary bra and ket states, with non-vanishing combinations being accepted. This calculational aspect does, however, not mean that any amplitude-level evolution algorithm is employed. Once we have also received comments from the second referee, we aim at including an formula that summarizes the splitting probability after the procedures described in sec II.B and II.C.

### Question III: What is the Sudakov operator?

The unitarity property of the parton shower relies on a consistent sampling algorithm, which is based on the colour-corrected real-emission pattern given by the splitting kernels. The reweighting procedure for fixed-colour states and matrix element corrections described in the paper allows for the correction of these emission patterns, while preserving the unitarity of the shower. We do not claim to do consistently exponentiated amplitude level evolution in this manuscript. Instead, the main aim of the manuscript was to assess if an  $1/N_c$ -improved description of real-emission patterns was phenomenologically meaningful without considering also QED effects, as argued in detail in the introduction.

The referee also comments on a Sudakov operator of their own definition. This definition assumes the application of a type of amplitude-level evolution. Since this is at odds with the methods and goals of the manuscript, we refrain from commenting on this issue.

## References

- [1] G. Gustafson, Nucl. Phys. B **392**, 251 (1993).
- [2] C. Friberg, G. Gustafson, and J. Hakkinen, Nucl. Phys. B 490, 289 (1997), hep-ph/9604347.
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- [4] S. Plätzer, M. Sjodahl, and J. Thorén, JHEP 11, 009 (2018), 1808.00332.
- [5] K. Hamilton, R. Medves, G. P. Salam, L. Scyboz, and G. Soyez, (2020), 2011.10054.