

Response to Report 2

Referee Comment: *When the authors say “there is a little mismatch between the dispersions close to the Dirac nodes”, it would be nice to be more precise. Is the mismatch in the meV range? Is it in the 10meV range? Is it larger? I think it would be best to show the band structure of the different models so that readers can judge by themselves.*

Our Response : We thank the referee for raising this point. The mismatch is around a few meV at the Dirac node due to λk_x^3 term. Since inclusion of this term will only complicate the four-band model without changing results quantitatively, in the revised manuscript, we only mention (first paragraph, left column, page 3) the mismatch value instead of adding another figure.

Referee Comment: *I am sorry I was not able to follow the sentence ‘Since the tight-binding model cannot not reproduce exactly the energy dispersion obtained from the DFT calculations anyway, we ignore such terms from now on’. Why the TB model cannot not reproduce the DFT results?*

Our Response : The point we tried to get across in the sentence itself is as follows: the discussed term (λk_x^3) allows to reach very good matching between the 8-band and 4-band models. At the same time, it complicates the 4-band model. Given that the 8-band model is not a perfect match to the DFT results anyway, including tiny corrections to the 4-band model does not improve its predictive power. The fact that the tight-binding model is not a perfect match to the DFT results is just an observation that is made in the references that derived the original 8-band model that we used. We added explicit citations to the text to emphasize this point. In the revised manuscript, we also added a follow-up sentence to clarify the very point of the sentence in question.

Referee Comment: *I am sorry I was not able to certainly understand this sentence: “the leading k -independent spin-orbit coupling gives rise to the conservation of the spin projection on a particular axis near the Γ -point, which is consistent with the sample, edge-orientation, and gate-voltage independent spin quantization axis on edges of a sample”. What is meant by “consistent with the sample, edge-orientation”? Does it means that it is at fixed angle away from the edge? Perhaps this sentence could be broken into peaces that makes its comprehension easier.*

Our Response : We already broke the sentence into two parts in response to the other Referee, hopefully it adds to its readability. By the sentence mentioned by the referee, we want to point out that the spin-quantization axis is independent of i) sample ii) gate voltage and iii) edge-orientation because it is fixed relative to the crystal structure. In other words, the spins of the edge states remain parallel or antiparallel to d_{so} , independent of the edge details as well as sample.

Referee Comment: *It could be useful to clarify why the Berry curvature appears in the field-induced Hall effect. Most readers will understand why it usually appears in the anomalous Hall effect (zero-field, by definition a different experiment).*

Our Response : Monolayer WTe_2 in its 1T' phase preserves both inversion and time-reversal (TR) symmetry. Therefore, the Berry curvature has opposite in sign, but equal in

magnitude for the degenerate bands of the material. Without a TR-breaking perturbation, there is perfect cancellation between the corresponding contributions to the anomalous Hall effect (AHE). An external magnetic field, splitting the degeneracy via the Zeeman effect, then yields a finite Hall effect. We added a clarifying sentence right after Eq.(25).

Referee Comment: *I don't understand why smaller spin-orbit coupling leads to larger Hall effect in Eq. 25. At least in the conventional AHC, in linear response, the AHC vanishes without SOC.*

Our Response : We have already addressed this question in our response to the other Referee. For convenience, we quote that response here:

“We note that we are talking about a situation near a band edge, where the gap - or the spin-orbital coupling - is the largest energy scale, so one could say that the Hall conductivity is inversely proportional to the gap, and that would be less surprising, perhaps. In more detail, the Berry curvature for the degenerate conduction bands in this system is given by (Eq. 24)

$$\Omega_z(\mathbf{k}) = -\frac{v_x v_y \Delta_{so}}{2\epsilon_{\mathbf{k}}^3} (\boldsymbol{\Sigma} \cdot \mathbf{d}_s). \quad (1)$$

In the low-energy limit, $\epsilon_{\mathbf{k}} \rightarrow \Delta_{so}/\sqrt{1-\beta^2}$ and therefore, the Berry curvature becomes $\Omega_z(\mathbf{k}) \propto 1/\Delta_{so}^2$.

To consider the limit of small SOC, one has to send the gap size to zero while keeping the Fermi level constant. Then for small enough gap, it will have only a perturbative effect on the Berry curvature at the Fermi level, and the AHE will simply vanish with vanishing SOC. This is not the situation we are interested in though.”