We thank the referee for raising important points and for their positive assessment of our manuscript.

We agree with the referee that a clear explanation of the units in Eq. 1 is necessary. Most importantly, the driving term of the two-level systems is expressed via an effective driving field strength E_d, which includes a coupling coefficient. This expression derives from semi-classical light-matter coupling as for instance in the Rabi-model, where the interaction term is proportional to the transition frequency w_z as well as the dot product of the driving field E with the dipole moment d of the atom. The corresponding product then relates to our effective driving field strength in this picture. In this case it is E_d = w_z E.d/2/hbar and then 2 E_d/w_d takes the role of the Rabi frequency. Another motivation for Eq. 1 is provided by the light-matter coupling in graphene, where the effective driving field strength is given by the product of the driving field strength with the elementary charge e and the Fermi velocity v_F at the Dirac cone. In this case it is E_d=e v_F E/hbar. We have included this explanation and these two models in the revision.

We agree that in the example in our manuscript the FSP emerges in the strong driving regime in which the Floquet energies scale linearly with the driving field strength E_d. Note that by setting the TLS energy scale omega_z closer to the driving frequency omega_d, and the cavity frequency omega_c closer to omega_z, population inversion and the resonance condition with Floquet energies are reached at lower driving field strengths within the quadratic scaling regime of the Floquet energies given in Eq. 18.

Consider the example omega_z=0.9 omega_d and omega_c=0.8 omega_d, which is not far from the example in our manuscript. Then the FSP onset occurs at roughly E_d,onset=0.087 omega_d^2. Then for a driving frequency of omega_d=2pi*48THz in the particular example of graphene, the light-coupling is given by the ratio of the Fermi velocity and the elementary charge which gives approximately E_d=5.2MV/m, which is very reasonable. In radio-frequency or microwave setups the scaling would be even more favorable. However, we agree that for larger driving frequencies, say far-ultraviolet, the scaling with omega_d^2 will lead to very large field strengths on the order of TV/m and larger, which may not be feasible, such that the FSP may be easiest realized in the sub-optical regime. In the revision we include a remark on this and a similar example.

In general we express the parameters in terms of the driving frequency omega_d. Since in our manuscript we use the example where omega_z=omega_d/2 and omega_c=omega_d/4, the Dicke superradiant transition occurs at lambda_c=omega_d/4/sqrt(2). Generally, in this setup these quantities are all on the same order of magnitude, which we believe makes it straightforward to relate lambda_c to physical units. Further, the FSP emerges at smaller values than lambda_c. In particular in a realization of the Dicke model where the Dicke superradiant transition is accessible, the FSP will also be accessible in terms of the coupling strength. Decreasing the cavity loss rate makes the FSP emerge at smaller coupling strengths, making it even more accessible. Roughly speaking a choice of lambda=omega_d/10 is appropriate for the FSP in examples that are comparable to ours.

Note that temperature enters our model in the expression of gamma_2 which is proportional to (1 - exp(-2 epsilon/(kB T))), where epsilon is the instantaneous eigenenergy of a given TLS. Epsilon will be on the order of omega_d, hence for driving frequencies that exceed roughly 2pi*20THz, this factor is equal to 1 for temperatures not exceeding room temperature. Therefore the FSP is sustainable at large temperatures in the THz regime. For smaller driving frequencies this ratio of energies will vanish at smaller temperatures, such that gamma_2 approaches 0, suppressing the FSP. Therefore smaller driving frequencies in the microwave or radio-frequency regime will require low temperatures making them less favorable than choices in the THz regime. With the previous argument on necessary driving field strengths, we conclude that tens of THz present the ideal driving frequency regime for realizing the FSP. We include this argument in the revision.

The dissipation in the instantaneous eigenbasis is a phenomenological approach that in previous work by Nuske et al. (Phys. Rev. Research 2, 043408 (2020)) has been shown to describe solids, in particular light-driven graphene, very well. The physical parameters in our manuscript are inspired by the driving frequency and dissipation coefficients in this previous work. Note that since the FSP is not very sensitive towards the TLS dissipation coefficients, we understand the FSP to be robust in a large parameter space of dissipation coefficients and temperature. It only vanishes in highly heated (kB T >> omega_d) or over-damped (gamma_- >> omega_d) cases.

The referee states concerns regarding the validity of the Lindblad master equation within these parameters, as they address the weak coupling (Born approximation) and the separation of time-scales (Markov approximation). Within the previously mentioned arguments, we believe that the Lindblad master equation in our system is well justified. The limitations of the Lindblad master equation are certainly intricate and important, especially for driven systems as explored by Teixeira et al. (New J. Phys. 24 013005 (2022)). While this work predicts deviations in using the Lindblad master equation in a system that is comparable to our work, they appear small enough that we feel confident in our method and our results regarding the FSP. We have included this reference and a remark in the revision. We leave a similar analysis as given in this reference of the FSP using a stochastic Liouville equation with dissipation to be performed elsewhere.