Reply to Referee 3

We thank Referee 3 for recommending publication of the article in SciPost. Thank you for useful suggestions and comments. These comments and suggestions have helped us to improve the manuscript and its readability. In the revised manuscript, we have clarified all the comments and questions raised by Referee 3.

1. **Q:** The analysis in page 3 can benefit from some physical explanations., for example, a term-by-term explanation of Eq. (3).
   Ans: Large part of the discussions on Matrix Product Ansatz is now pushed to a separate section, "APPENDIX". A term-by-term explanation of the Master equation and how it leads to Eq. (13) is discussed.

2. **Q:** Are the simulations for obtaining F_r, F_l and F performed in the steady state? Then are the two urns (or particles) chosen at random? Or do the authors choose specific initial states?
   Ans: The simulations for obtaining F_r, F_l and F are performed after relaxing the system for sufficiently long time so that it is in the coarsening regime (now mentioned in the text). If true condensation (phase separation) exists, then the system may take a very long time to reach there, but the dynamics in the coarsening regime can predict, well in advance, if the system is approaching towards MIPS or a homogeneous state (now it is mentioned in the revised manuscript).

3. **Q:** What are the functional forms of η_+ and ρ_+ in terms of γ?
   Ans: Explicit derivation and the functional form of η_+ and ρ_+ are now given in the APPENDIX.

4. **Q:** Can the authors comment on some transient properties? For example, if we had an initial state of a given density with all particles being placed on consecutive lattice sites and the rest empty, i.e., we start with a macro-cluster. Is there a (heuristic) time-scale for this cluster to break down into a state of globally homogeneous density? I expect the transient states to depend on the hopping and tumbling rates, as well as on the density of particles. Can this line of thought give any insight for the non-existence of MIPS in the model?
   Ans: In a periodic homogeneous system, statistical average of density profile would be flat. Thus to study the transient properties either one takes a very large system that evolves for very very long time (which is computationally heavy) or we need a very careful study of small systems taking care of the finite size effects. The particular regime of interest, $\omega \to 0$ limit, has additional complications as ergodicity is broken at $\omega = 0$ and system there falls into one of the fully jammed/absorbing configurations. The lifetime of the transient states will surely depend on the hopping/tumbling rates and the density as you mentioned but such numerical studies are always surrounded by questions. For example, AHR model (Ref. [25]) exhibited phase separated states in accessible laboratory time-scales but it was proven later (Ref. [24]) that a phase separation transition is not possible there. An analytical result, though at coarse-grained level, is useful.

   We do understand that lifetime of the transient states is a very important question, but we leave it for future studies because a definite and conclusive statement about non-existence of MIPS can not be obtained from these studies in a short time.

5. **Q:** The authors assume that the hop rate $u(m_k, m_{k+1})$ depends on the number of particles in the departure site k and the arrival site $k+1$. Can they elaborate on the insight behind this assumption? Should the hop rate also not depend on the internal states of the arrival and departure urns? Naively, one might assume the current within a domain to depend on the polarities of the particles at the end of the domain.
   Ans: RTM can be mapped exactly to a urn model where urns have internal degrees (spins) and the hop rates depend only on the spins of arrival and departure urn not on the number of beads (as shown in new Fig1(a)). We then provide a coarse-grained description in Fig1(b) by integrating the spin of arrival and departure urns. Now, the urns do not carry spin index but the hop-rates acquire dependence on the number of beads present in arrival and departure urn. Mapping the model to misanthrope process instead of zero range process is relevant as the dynamics of the original urn model was dependent on the properties (spin) of both arrival and departure urns.

6. **Q:** Can this restricted tumbling model show or rule-out micro-phase separation with formation of stable micro-clusters in the steady state?
   Ans: Since distribution of beads exhibit exponential form for both (i) the coarse-grained urn model (see Fig

Fig2(b)) and (ii) approximate matrix product state ($p_+(m) = y^m/F(y), p_-(m) = \gamma^m y^m/F(y)$, as mentioned after Eq. (19)), it is unlikely that there are 'stable' micro-clusters in steady state. But honestly, we do not know, how to show or prove the existence of any 'stable' micro-clusters.

7. **Q:** Indeed the manuscript could benefit from citing further recent literature on interacting one-dimensional run-and-tumble systems.
   Ans: References [15], [16] and [23] are added.

8. **Q:** Kindly address the points mentioned in the report and correct typos, e.g., 'sate' $\rightarrow$ state ,'dependn' $\rightarrow$ depends.
   Ans: These typos are corrected.

Summary of changes

The article has gone through a **major revision** following the valuable comments of the referees. It is difficult to provide details as in the revised manuscript, the whole structure and almost all the paragraphs are modified. In the following we list some important changes.

1. Restricted tumbling dynamics is now given separately as Eq. (2) followed by a longer discussion.

2. Fig. 1 is modified - exact and coarse-grained urn models are now described more clearly in Fig1(b).

3. Discussions on Matrix Product Ansatz (MPA) is described in the APPENDIX. We hope it helps the readers to arrive at the results and conclusions of the article without bothering much about the detailed mathematical steps of MPA.

4. New references [15], [16] and [23] are added.