

We thank the referee for the careful reading of our manuscript and making several important remarks which have helped us to improve our presentation. We also thank the referee for pointing out several strengths of our work and finding our paper to be well written and addressing timely topics.

We would first like to address the referee's concerns regarding the novelty of our work and how this work goes beyond the results discussed in the existing literature.

## I. IMPORTANCE OF OUR WORK

We present a list of points about the novelty of the findings reported in our paper.

- 1. Dynamical localization and interactions:** The referee has correctly pointed out that dynamical localization (DL) in a single-particle model is a well-studied phenomenon in various periodically driven systems. However, DL occurring in the large driving amplitude regime in the class of Floquet models proposed by us (with spatially periodic potentials which are driven in time) has not been discussed in the literature to the best of our knowledge. Moreover, it has been argued before that DL can be completely destroyed in the presence of interactions (Luitz et al, SciPost Phys. **3**, 029 (2017)). However, our work indicates that this statement is not quite generic. In fact, our proposed models demonstrate many-body flat bands in the presence of interactions at DL points, which points toward the stability of DL even when interactions are present.
- 2. Dynamical localization and weak interaction limit of the period-4 model:** In our work, we have studied two mirror-symmetric classes of the period-4 model in detail. These two models at the DL points behave in entirely different manners. While the period-4 model with  $\phi = 0$  reduces to a period-2 model at a DL point, the model with  $\phi = 7\pi/4$  appears to be much more intricate. The model at  $\phi = 7\pi/4$  in the presence of weak interaction at a DL point reduces to the interacting Su-Schrieffer-Heeger model (SSH model). Remarkably, we have discovered that this SSH model with interactions turns into the transverse field Ising model (TFIM) with or without boundary fields under an effective spin mapping, and this model is exactly solvable. Such a spin mapping of the interacting SSH model has not been reported before in the literature as far as we know. We have further shown the existence of many frozen state configurations due to the intricate interplay of DL and interactions; this again indicates the stability of DL in the presence of interactions.
- 3. Dynamical localization, resonance, and strong interaction leading to Hilbert space fragmentation:** We agree with the referee that there are a plethora of equilibrium models proposed in recent years where Hilbert space fragmentation (HSF) occurs due to the presence of kinetic constraints. However, HSF arising in a periodically driven model due to an interplay among DL, resonance, and strong interaction has not been seen until now, to the best of our knowledge. Therefore, our proposed models are the first where such phenomena have been observed, and we have supported this by a number of numerical and analytical arguments as also pointed out by the referee. Moreover, one of the most well-studied time-independent models exhibiting HSF satisfies the conservation of total particle number and total dipole moment, and it arises in the large interaction limit of an equilibrium model (Tomasi et al, Phys. Rev. B **100**, 214313 (2019)). Although their model looks similar to the effective Floquet model of HSF discussed in our work, the two are not identical. The main difference is that our effective Floquet model conserves the total particle number and a staggered Ising interaction, but not the total dipole moment. We also note our model does not arise in any limit of an equilibrium model. The fact that our model exhibits HSF is a new result.
- 4. Experimental accessibility:** Flat-band induced quantum many-body scars and HSF have been observed in recent years in the context of equilibrium systems. One of the common mechanisms for these features is compact localization, which requires special kinds of lattice structures. However, DL-induced flat bands can appear in extremely simple lattice models; therefore, this mechanism is experimentally more accessible compared to systems demonstrating compact localization. This is one of the main reasons that motivated us to pursue this idea in the first place. Moreover, the intricate interplay between dynamical localization and resonance induces a special kind of Hilbert space fragmentation (HSF) in our model which is different from the HSF models studied in the literature so far. Further, the kind of driving we have chosen for our proposed models can be realized in cold atom systems, and therefore, opens up new possibilities for further explorations.
- 5. Interplay between drive-induced dynamical localization and interaction leading to various mechanisms of ergodicity breaking being observed in a single model:** We respectfully disagree with the point (2) mentioned by the referee. It is true that single-particle DL, drive-induced quantum many-body scars, HSF, and drive-induced weak integrability breaking have been separately found in various models studied in the literature. However, the class of models proposed by us appears to be the first where all these phenomena can be simultaneously observed in a simple periodically driven and experimentally accessible lattice model due to an interplay between DL and interactions. In our opinion, therefore, our results go well beyond those reported in the existing literature.

We hope that the points mentioned above clarify the novelty and importance of our work.

## II. REQUESTED CHANGES

1. The different panels in Figure 1 all show essentially the same except for a different energy scale. There is no need for three panels but the change in scale can be explained in the caption.

We thank the referee for this comment. We have now removed Fig. 1 (b). However we have kept Fig. 1 (c), which carries information about the change in the bandwidth due to the third-order correction which become more important as the value of  $\mu$  is decreased.

2. Fig.4 c upper panel does show long-lived revivals but there is a clear decay. What is the functional form of the decay and how does it depend on the frequency and parameters?

We thank the referee for this question. We have now added two figures, Figs. 23 (c) and (d), where we have numerically fitted the envelop of the Loschmidt echo for two sets of parameter values, namely,  $\mu = V = \omega = 20$  and  $\mu = V = \omega = 10$ . We have extracted the decay rate from the numerical fitting of the envelop, and have discussed the possible relationship between the decay rate and the parameter values, as asked by the referee.

## III. LIST OF CHANGES MADE IN THE REVISED MANUSCRIPT

We have shown the major changes in blue in the revised manuscript.

1. We have added two sections, namely, the thermodynamic stability of Hilbert space fragmentation in this class of models (Sec. 5) and the experimental accessibility (Sec. 6).
2. We have added the derivation of the third-order effective Hamiltonian at the dynamical localization point in Appendix C. This is relevant for answering a question asked by the first referee.
3. We have removed Fig. 1 (b). However we have retained Fig. 1 (c), which is important for seeing the variation of the bandwidth due to the third-order corrections which become larger when the value of  $\mu$  is decreased.
4. We have added two figures (Figs. 23 (a) and (b)) showing the variation of the Loschmidt echo with time for the resonant case of the period-2 model at a dynamical localization point for two sets of parameter values. We have numerically fitted the envelop of the Loschmidt echo varying with time to extract the decay rate of the envelop and have discussed how the decay rate is related to the parameter values used.
5. We have added a figure (Fig. 2 (e)) showing how the crossover scale  $n_c$  diverges as one approaches the critical frequency  $\omega_c$  from the  $\omega > \omega_c$  side. This further confirms our analytically derived result.
6. We have added a figure (Fig. 5 (c)) showing a plot of  $E_{exact}$  vs  $E_{FPT}$  for the resonant case of the period-2 model at a dynamical localization point. We have fitted the plot to quantify the agreement of the first-order Floquet perturbation theory with the numerically obtained results.
7. We have mentioned certain symmetries of our effective Floquet Hamiltonian obtained for the period-2 model at resonance and at a dynamical localization point to contrast this kind of Hilbert space fragmentation (HSF) with the models showing HSF which were known earlier.
8. We have corrected some typos pointed out by the referees.