Response to Reviewer's Comments

Title: Matter Wave Isotope Separation in a Ring Trap

We would like to extend our sincere thanks to the reviewer for appreciating the importance and experimental relevance of our work and also for giving important suggestions. We have made the appropriate changes in the revised manuscript as per the Reviewer's comments.

- **Reviewer's Comment**: *First, it is correctly indicated that effective coefficients of the nonlinear interactions can be adjusted by means of the Feshbach resonance (FR). Then, it will be quite relevant to produce a parameter chart similar to Fig. 6 in which one control parameter will be the strength of the magnetic field which controls the FR.*

Reply:

We concur that including magnetic field values corresponding to the interspecies interaction strength will be more physically intuitive and it will enhance the relevance of the work in terms of experimental realization. To address this aspect, we have prepared a table (see below), which is also included in the revised manuscript.

The white dotted line in Fig. 6 of the revised manuscript suggests physical parameters for highest isotope yields for experimental feasibility which are listed in the Table as follows:

$r_0(a_\perp)$	$a_{12}(a_{11})$	$B(G)$	S_1 Separability (%)
8	0.3	259.17	92.8
9	0.28	259.20	96.1
10	0.3	259.17	97.2
11	0.32	259.14	98.4
12	0.32	259.14	98.9
13	0.3	259.17	99.0
14	0.3	259.17	98.8
15	0.18	259.36	99.1
16	0.12	259.45	97.4

Here, S_1 separability is evaluated for a given radius and external magnetic field. The external magnetic field **B** controls the interspecies interaction strength a_{12} through the Feshbach Resonance:

$$a_{12} = a_{bg}\left(1 - \frac{\Delta B}{B_0 - B}\right)$$

Here, the background scattering length is taken as $a_{bg} = 11.27 \times 10^{-9}$m, Feshbach resonance peak is $B_0 = 265.42$ G and its width is taken as $\Delta B = 5.8$ G following reference [1].

- **Reviewer's Comment**: *Next, it would be quite relevant to compare the results reported in the paper for the 2D model with what can be produced by the 1D equations in the limit of an infinitely thin ring. I assume it is not too difficult to add some results of the simulations of the 1D system.*

Reply:

We appreciate this suggestion for seeing it in the limit of infinitely thin ring. As our proposal for isotope separation is based on the phenomena, called fractional revivals, we have performed a numerical simulation for a very thin ring where a quasi-1D BEC is considered. We have observed fractional revivals in this case too with certain points in contrast to the isotope separation in the 2D ring case:

a. The revival and fractional revivals timescales of the two species merge relatively faster than the 2D case.
b. The merging of timescales results in inferior spatial separation of isotopes, which is illustrated in the following figure by showing the frequency of occurrence (above 90% separability) in quasi-1D and quasi-2D cases.
c. It is broadly observed from the figure that the probability of 'above 90% separability' is lesser in quasi-1D cloud compared to the quasi-2D one (compared by the number of points in the following figure).

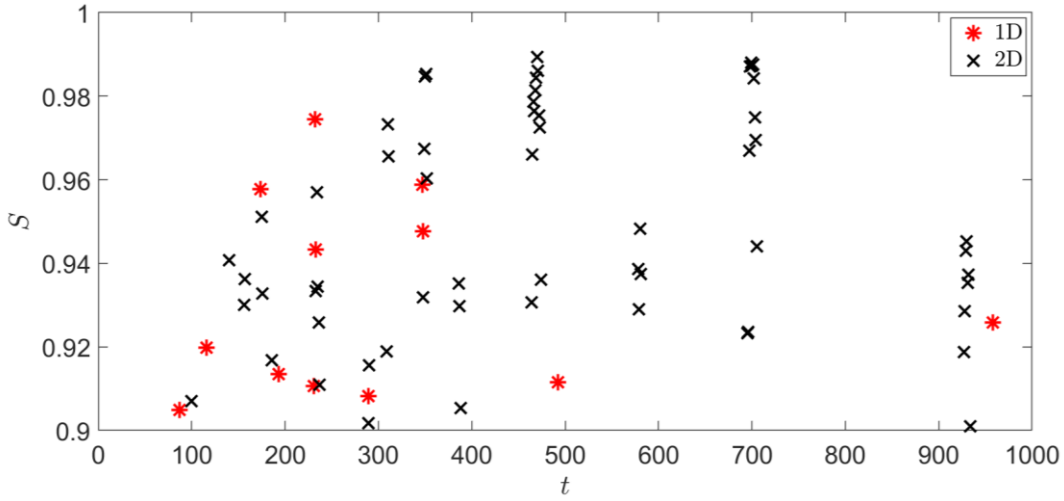

Figure 1 Occurrences of Separability above 90% with time for 1D and 2D models.

We have added a subsection in the revised manuscript regarding the above analysis.

- **Reviewer's Comment**: *Finally, as an optional recommendation, it might be quite interesting to extend these results for the case when vorticity imparted onto one or both components of the mixture.*

Reply: We are thankful to the reviewer for providing an insightful recommendation. As far as matter wave vortices in a ring trap are concerned, the 1D and 2D ring traps can deliver different types of vortices [2]. While the 1D ring supports no visible vortex [3,4], the 2D ring can accommodate both hidden and visible vortices [5,6]. Investigating the revival dynamics of a localized Bose-Einstein Condensate (BEC) featuring a visible vortex within a 2D ring trap, through very nontrivial, is an intriguing problem in itself. As the incorporation of any study in the present manuscript will deviate the main focus of the work in the context of isotope separation, we would like to try solving this interesting problem in future with due credit to the reviewer by exploring the impact of fractional revivals of matter wave packets in both a rotating ring and a hidden vortex-imprinted ring trap.

References

1. Papp, S. B., and C. E. Wieman. "Observation of heteronuclear Feshbach molecules from a Rb 85–Rb 87 gas." *Physical Review Letters* 97, no. 18 (2006): 180404.
2. Hejazi, S. Sahar S., Juan Polo, and Makoto Tsubota. "Formation of local and global currents in a toroidal Bose-Einstein condensate via an inhomogeneous artificial gauge field." *Physical Review A* 105, no. 5 (2022): 053307.
3. Gajda, M., M. Lewenstein, K. Sengstock, G. Birkl, and W. Ertmer. "Optical generation of vortices in trapped Bose-Einstein condensates." *Physical Review A* 60, no. 5 (1999): R3381.
4. Polo, J., Dubessy, R., Pedri, P., Perrin, H., & Minguzzi, A. (2019). Oscillations and decay of superfluid currents in a one-dimensional Bose gas on a ring. *Physical Review Letters*, *123*(19), 195301.
5. Piazza, F., L. A. Collins, and A. Smerzi. "Vortex-induced phase-slip dissipation in a toroidal Bose-Einstein condensate flowing through a barrier." *Physical Review A* 80, no. 2 (2009): 021601.
6. Mathey, Amy C., and L. Mathey. "Realizing and optimizing an atomtronic SQUID." *New Journal of Physics* 18, no. 5 (2016): 055016.

List of Changes in the Manuscript (**Matter Wave Isotope Separation in a Ring Trap)**

1. A Table with appropriate explanation has been included in the revised version of the manuscript for illustrating the values of ring radius, interspecies interaction strength and the corresponding magnetic field values that support maximum $\mathbf{S}_1$ Separability. Also, the experimental values of Feshbach Resonance peak position, width and background scattering length are provided.
2. A subsection about the Isotope Separation in Quasi-1D BEC trapped in an infinitesimally thin ring is included in the revised version as per Reviewer's comment.
3. One subsection, One table and One figure are included in the revised manuscript to address the above points.