Dear Editors,

we thank the referees for their helpful reports, and apologize for the lengthy "awaiting resubmission" status of our manuscript on the SciPost submission web site. We have finally revised the manuscript heeding the advice of the two referees.

We reply to their suggestions, questions, and criticism below. The referee reports are *italicized* and our replies are non-italicized.

Reply to Report of the 2nd Referee

Strengths

- the introduced tHNC method has computational advantage over existing approaches
- the approximations of tHNC are clearly stated and ways for further improvement of the method are outlined
- tHNC is used to tackle interesting and hard non-equilibrium many-body problems

Weaknesses

- the range of the validity of the approach is not fully clear
- some minor aspects of the presentation could be improved

Report

The authors develop a tHNC method for non-equilibrium many-body problems and employ it in investigations of quench dynamics of Bose gases. The method is presented in a clear and well-justified manner, with emphasis on the underlying assumptions. Application of the tHNC method to 3D gas of Rydberg-dressed atoms yields insights about interaction quenches in the system, and the authors propose plausible explanations for the observed phenomena. Comparison of tHNC method with tVMC for 1D and 2D systems allows to obtain certain intuitions about the range of validity of the introduced approach.

The novel method proposed in the present manuscript allows for non-perturbative insights into dynamics of "not too strongly correlated" many-body systems. This hard problem is both conceptually interesting and of practical importance. Therefore, I recommend that the manuscript "Interaction quenches in Bose gases studied with a time-dependent hypernetted-chain Euler-Lagrange method" be published in SciPost Physics once the authors address the following minor remarks.

1. The tHNC method does not require Monte Carlo sampling and hence it is significantly faster than the tVMC. It would be helpful if authors included some estimates of computational resources needed to perform the simulations from Sec. III, both for tHNC and tVMC.

> The tHNC method is indeed orders of magnitude faster than tVMC (while using more approximations). Even though our tVMC code is parallelized and scales well up to hundreds of CPU cores, it requires a significant amount of computation time. We have added a comparison of the total number of CPU hours required by tHNC and tVMC for the simulation of one- and two-dimensional systems in appendix B.

2. Data shown in Fig. 6 and 7 indicate that tHMC gets less reliable for larger interaction amplitudes, especially for longer times. How can we be sure that data presented in Fig. 2 for the larger values of R/r0 and times up to t/t0=20 (or for t/t0=40 in Fig. 3) are in any correspondence with the actual dynamics of the system?

> We cannot be sure. Only for discrete many-body Hamiltonians in 1D, there are essentially exact methods to compare with. For continuous Hamiltonians and especially for more than one dimensions, neither tVMC nor any other method will produce the actual dynamics of the system. Since tVMC has fewer approximations, we expect results closer to the actual dynamics. Due to the high computational cost we refrained from 3D tVMC simulations, where we show only tHNC results. Indeed, the comparison for 2D (for the Rydberg potential as in 3D) in Fig.7 shows that for longer time t tHNC deviates from tVMC for $g(r,t)$ at small distance r. This means that the oscillations of $g(r=0,t)$ in Fig.3 are probably not reliable for large t anymore. This is also demonstrated in Fig.8 comparing such oscillation in 1D with tVMC: there, the deviation between tVMC and tHNC become quite large for large t, but it should be noted that correlations are much stronger in 1D than in higher dimensions; we show the 1D comparison as a worst case scenario.
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> Regarding the reliability of the approximations behind the tHNC method (Jastrow ansatz and no elementary diagrams) we refer to our new appendix C; for details, see our reply to your question 5).

3. Page 6. the meaning of ”nonlinear process” is not clear but is important to understand the reasoning of the authors.

We mean that such a quench of the interaction range R by more than a factor of 4 cannot be regarded as a small perturbation that can be treated with linear response theory. The quench $R = 4r_0 \to 4.5r_0$ shown in the inset, on the other hand, is more amenable to linear response theory, as the relative change of R is much smaller. There we observe an oscillation of $g(r,t)$ much closer to twice the roton frequency, which is what we would expect from linear response theory.
We added a corresponding explanation in the text.

4. Can the authors provide an explanation for the suppression of the excitation of two maxons during the 4r0→4.5r0 quench, perhaps due to energetics of the system?

This is a good question. The quench $R = 1r_0 \to 4.5r_0$ is much more violent than $R = 4r_0 \to 4.5r_0$, therefore depositing more excess energy into the system, i.e. in addition to the energy that an adiabatic increase to $R = 4.5r_0$ would require to simply raise the ground state energy. The excess energy of the strong quench is more than 10 times larger than that of the weak quench. This makes it less likely to excite a maxon, which has more than twice the energy of a roton at $R = 4.5r_0$.

We have added this explanation in the manuscript, also providing the precise values of the excess energy for the weak and strong pulse.

5. The authors mention mean field studies of quench dynamics in Rydberg dressed Bose gases [34-36]. For the sake of self-completeness of the manuscript, it would be useful to contain a more in-depth comparison of the results of tHNC with the earlier simpler approaches.

Since its not possible to be sure which method is the most reliable (mean field based on the Bogoliubov approximation, tHNC or tVMC), because there are no exact results to compare with, we decided to do the following: from previous work by some of us, we have ground state results (to be precise, low temperature results) for the Rydberg-dressed Bose gas from exact path integral Monte Carlo (PIMC) simulations. We compared the exact $g(r)$ with that obtained from the ground state HNC-EL (using the same Jastrow-ansatz and omitting elementary diagrams as in tHNC), and with two variants of the Bogoliubov approximation used previously for dynamic studies ([34-36] in the old manuscript). We present these results in a new appendix C.

We find that at least in the limit of no dynamics where tHNC becomes HNC-EL, our method is reliable. $g(r)$ obtained with HNC-EL agrees very well with the exact results, while both variants of the Bogoliubov approximation are far off the exact result.

6. In the introduction section, the addition of references to TDVP (alongside of TEBD) and some standard works on BEC, quench experiments, Feshbach resonances would help make the description more concrete and provide a better context for the study.

We expanded the introduction with general remarks on quantum gases and specific examples of experimental non equilibrium studies with ultracold quantum gases, including new references (see end of this reply). We moved up and rewrote the paragraph on quenches with Feshbach resonances, and we added references for the time-dependent variational principle and its application to simulations with matrix product states.

List of Changes:

1. We added a comparison of the computational effort of tHNC and tVMC.
2. We explain what we mean by “nonlinear process”
3. We provide a comparison between the Bogoliubov approximation, the HNC approximation and the exact result for $g(r)$ in the ground state in a new appendix C, including a new Fig.8.
4. We improved Fig.2 and its caption: adding a color bar, adding the sound cone and explaining the color scheme;
5. At the start of the results sections we elaborated a bit more on how the result section is structured
6. At the start of section 3.1 we give a physical picture of the Rydberg-dressed interaction.
7. We moved the technical details of the tVMC simulations to the appendix.
8. We extended the discussion of the “light cone” ideas considerably, describing and explaining the additional light cone lines in Fig.1.
9. We combined former Fig.1 and 4 into one Fig. with two panels, because in the new discussion of the light cone we need to refer to Fig.4 earlier in the text than in the old manuscript.
10. We updated Fig.7 (former Fig.8) by showing the stochastic error.
11. We converted the inline equation for the action S into equation (4) in order to reference this definition in app.A
12. In app. A, we explicitly show the time dependence of all quantities instead of omitting it
13. We added a reference for Feshbach resonances in the introduction
14. various small changes in formulation

New References:

1. A. Kerman and S. Koonin, Annals of Physics 100, 332–358 (1976).
2. Haegeman et al, PRL 107, 070601 (2011)
3. G. Pupillo et al, PRL 104, 223002 (2010)
4. C. Chin et al, Rev. Mod. Phys. 82, 1225 (2010)
5. L. Madeira and V. S. Bagnato, Symm. 14, 678 (2022)
6. F. Dalfovo, S. Giorgini, L. P. Pitaevskii and S. Stringari, Rev. Mod. Phys. 71, 463 (1999)
7. O. Morsch and M. Oberthaler, Rev. Mod. Phys. 78, 179 (2006).
8. A. Polkovnikov et al., Rev. Mod. Phys. 83, 863 (2011)
9. T. Langen, R. Geiger and J. Schmiedmayer, Annu. Rev. Condens. Matter Phys. 6, 201 (2015)
10. M. Cheneau et al., Nature 481, 484 (2012)
11. P. Makotyn, Nat. Phys. 10, 116(2014)
12. N. Navon et al.,Nat. Phys. 17(12), 1334 (2021)