List of changes

Sec. 2

[1] We added a new subsection Sec. 2.3, titled "General remarks on the QRP".

[2] 1st paragraph in Sec. 2.3: We added a discussion clarifying our choice of zero-delay tasks and highlighting how finite-delay tasks can probe memory-related phenomena, including ergodicity.

[3] 1st paragraph in Sec. 2.3: We added descriptions to emphasize the importance of the flexibility to choose read-out operators.

[4] 1st paragraph in Sec. 2.3: We added a sentence introducing the applicability of the QRP to quantitative analysis, such as perturbative effects.

Sec. 4

[5] 3rd paragraph in Sec. 4: We modified the discussion about the QRP configurations according to the addition of Sec. 2.3.

Others

[6] We refined the phrasing for improved clarity and precision.

[7] We added a reference [51].

Reply to the referee #2

The authors build upon quantum reservoir computing (QRC) paradigms, proposing an inverse approach that exploits QRC dynamics as a probing tool to detect properties of quantum many-body systems. After outlining the general features and properties of QRC, the authors introduce a protocol that aims to reconstruct the value of a random input number via linear regression of dynamical observables, as usually done in reservoir computing. The idea of the work is certainly interesting, however, there are several aspects that, in my opinion, need to be improved in order for the manuscript to be accepted for publication in SciPost Physics.

We are very grateful to the reviewer for reading our manuscript carefully. We deeply appreciate his/her comments, "*The idea of the work is certainly interesting*". He/She raised several questions and suggestions in his/her report, which we have addressed point by point below. Accordingly, the manuscript has undergone a major update, most notably with the addition of a new Section 2.3 (see the summary of changes [1]).

First of all, I see many qualitative observations (if the dynamics of systems in different phases are different, it is not surprising that the computational performance looks different). While the authors mention that such performance can be used as a metric, I fail to see it.

We agree that a change in computational performance between two distinct phases might appear obvious. However, the novelty of the QRP is not this observation itself, but the fact that one can leverage that performance difference to extract insights into quantum many-body physics.

In this work, we have applied QRP specifically to the study of information propagation. As shown in Fig. 5, QRP faithfully reproduces the hallmark features of propagation dynamics revealed by traditional measures. More importantly, our approach offers concrete advantages: it exhibits heightened sensitivity to small perturbations and directly uncovers the information propagation channels. These observations demonstrate that the performance in the QRP is not only reliable but also uniquely powerful as a metric for information propagation.

As the reviewer pointed out, our current analysis emphasizes qualitative insights, as is standard in studies of information propagation, where, for instance, the negativity of TMI typically matters more than its absolute value. Nonetheless, the quantitative aspects of the performance in the QRP could also provide valuable insights when a more nuanced or quantitative understanding of underlying physics is required. For example, a systematic investigation of how QRP performance quantitatively varies with perturbation strength would yield deeper insights into perturbative effects. Performance metrics, such as mean squared error and mean absolute error, would be particularly useful in facilitating such quantitative evaluations. A detailed exploration of these quantitative aspects remains an intriguing direction for future research.

We added such descriptions in the revised manuscript. (See the summary of changes [4].)

Furthermore, the protocol seems to rely on a systematic scan of different read-out operators, which is not necessarily an efficient method.

The power of the QRP lies precisely in its ability to flexibly investigate a quantum system through a variety of read-out operators. As Figs. 5 and 6 show, each choice of operator produces a distinct qualitative and quantitative signature that directly reflects internal features of the Hilbert space. Fixing the read-out operator inevitably erases this fine-grained resolution, and with it the deep insights into information propagation that QRP is specifically designed to uncover. We note that conventional metrics remain blind to such operator-level propagation dynamics. Although systematically scanning multiple operators may seem inefficient, the unique depth of understanding QRP provides more than compensates for any efficiency trade-off.

We added such descriptions in the revised manuscript. (See the summary of changes [3].)

In most of the presented results, the system is employed as an extreme machine learning model—specifically, for zero-delay tasks where memory effects are irrelevant. However, the presence or absence of memory in the dynamics is a fundamental property that distinguishes ergodic and nonergodic phases in quantum systems. This critical aspect is entirely overlooked in the manuscript, despite its direct relevance to both quantum many-body physics and reservoir computing performance. A discussion of how memory effects—or their absence—manifest in the proposed protocol would significantly strengthen the work, particularly in clarifying whether the method probes purely instantaneous properties or can also capture time-dependent correlations in the quantum system.

We emphasize that the primary goal of this paper is to investigate information propagation in quantum systems using the QRP, a topic whose significance is well-established within quantum many-body physics. We chose the zero-delay task specifically because it aligns optimally with this objective; in other words, our targeted phenomena do not involve memory effects, which is why we did not prioritize higher-delay tasks.

As the reviewer noted, memory effects have indeed proven valuable for exploring certain aspects of quantum many-body physics, such as distinguishing between ergodic and nonergodic phases. If the phenomena of interest were related to such memory effects, employing QRP with non-zero delay tasks would offer meaningful insights. In fact, as demonstrated in Fig. 3, QRP can effectively handle tasks involving delays $d > 0$. However, given that our focus in this paper is centered explicitly on phenomena independent of memory effects, we consider a detailed analysis of the relationship between the QRP and memory properties beyond the scope of our current investigation. Such analysis is thus deferred to future research.

We added such descriptions in the revised manuscript; see the summary of changes [2].

The study focuses exclusively on two specific points in the parameter space—one associated with chaotic dynamics and the other with ballistic information propagation—both of which are well-characterized in prior literature. While this approach provides a useful starting point, it leaves several critical questions unanswered. Most notably: Phase Boundaries and Criticality Generality Across Parameter Space Broader Implications for Critical Phenomena: If, as I suspect, the protocol performs well near critical points, this could open new avenues for data-driven detection of quantum phase transitions.

We greatly appreciate the valuable suggestion regarding the potential application of the QRP for detecting quantum phase transitions. While this manuscript cannot exhaust every potential application, follow-up studies are already extending the scope of the QRP. Notably, a recent study posted roughly half a year after this manuscript (arXiv:2402.07097; to appear in Nature Communications) has successfully employed QRP to identify quantum phase transitions, exactly as recommended. We added this citation; see the summary of changes [6].

We are confident that QRP's versatile capabilities will inspire further exploration across diverse contexts and that the insights presented in this paper will serve as a solid foundation for future research.

In summary, while the proposed inverse QRC approach is innovative and theoretically intriguing, the study in its current form requires substantial strengthening to meet the standards for publication in SciPost Physics.

We significantly enhanced the manuscript so that the ideas and conclusions are well understand. We hope that the reviewer will recognize the important contribution of our study and recommend the publication of the revised manuscript.