Response Letter

Dear Editors,

Thank you for reviewing and handling our submission 'Collective dynamics of densely confined active polar disks with self- and mutual alignment'(manuscript ID: scipost_202501_00037v1).

We are resubmitting a revised version, following the suggestions by the reviewers. We have highlighted our changes to the manuscript in red, and our responses below in blue.

We believe that our manuscript has greatly improved as a result of the reviewers' feedback. All comments, questions, and criticisms have been addressed below. We thus respectfully request that our manuscript be accepted for publication in Scipost.

On behalf of all the authors,

With regards,
Pawel Romanczuk and Cristián Huepe

Reply to Report by Referee 1

Comments to the Author:

I read the manuscript 'Collective dynamics of densely confined active polar disks with self- and mutual alignment" by Weizhen Tang and co-workers. The article presents two-dimensional numerical simulations of self-propelling polar disks in a circular confinement. The main goal is to understand the role of alignment in the emergence of specific collective states. This is done cleverly by controlling the distance R between the geometrical center and the center of rotation of the disks, since mutual alignments become prominent as R increases. The roughness of the circular wall and type of damping (isotropic vs. anisotropic) are also considered. The simulations reveal that milling vortices form when mutual alignments are important and/or in the presence of smooth circular confinements.

The numerical results are interesting and represent a significant advance compared to, e.g., Refs. 40-41. However, before recommending publication I suggest the authors to improve their presentation. As several parameters (roughness of the boundary, R, type of damping) affect the collective states, it would be useful to include a Section in which they are considered one by one. For example, the authors could begin from the case [R=0.03, isotropic damping, smooth boundary] and illustrate what happens when one of the parameters changes.

Response: We thank the reviewer for these suggestions. We have added, in general, a more systematic exploration of the parameter space, to address the questions and issues raised by both referees. In particular, we have now included phase diagrams showing P and M as a function of R and D_θ, for all combinations of isotropic or anisotropic agent-substrate damping, and of smooth or rough boundary, in Fig. 18 of Appendix **A**.

Other remarks:

1) Eq. (2) includes white noise, but it does not seem to be relevant for any of the following results. If this is the case, wouldn't it be better to neglect it entirely? It is awkward to me that rotational noise is considered, but the translational one is not. Also, the diffusion coefficient D_θ is not defined.

Response: We thank the reviewer for these comments, since our motivation for including the noise parameter was indeed not clear in the previous version of the manuscript. Now that we have added the D_θ–R phase diagram mentioned above, it is clearer that the D_θ term is added to show that all the presented dynamics can be reached for a range of (relatively low) noise levels, and could thus emerge in real-world conditions. We have now stated this explicitly at the end of the new Appendix B, where we display the new phase diagrams. Regarding the use of only rotational noise, we did this to add noise in a minimal way while following the most common approach in self-propelled systems (i.e., the most used in the literature that followed the introduction of the Vicsek model, which only considered rotational noise). Finally, we have now properly defined D_θ in the manuscript, after Eq.(2).

2) I would increase the font of the symbols in Fig.1a.

Response: We have increased the font size for the symbols in Fig.1a to improve readability.

3) It would be instructive to see a plot where M (degree of milling) and P (polarization) are plotted as a function of R. For this, a few additional simulations for intermediate values of R are required.

Response: Following the reviewer's request, we have now added a new Fig. 10 to Appendix B (where the new phase diagram is also displayed), showing the polarization P and degree of milling M as a function of R

4) Page 8, "We note, in addition, that the amplitude .... when compared to their isotropic counterparts." Where does the reader see this in Figure 3.

Response: We are referring to the fact that the amplitude of the red curves in Figure 3 becomes smaller at a faster rate in the anisotropic case (bottom panels) than in their isotropic counterparts (top panels). In order to clarify this point, we have now extended this sentence to specify this.

5) In all Figures the axes do not have units. Can the authors clarify?

Response: We did not include units in the figure axes because all our results are expressed in the natural units stemming from the equations of motion (1-2) and from the parameter choices in Table 1, where all these parameters and their values are presented. For example, given that we set $l_0 = 1$ our length unit is given by the diameter of the disks. In order to clarify this, we now have added a comment at the end of Appendix A, after introducing Table 1.