Comment on "Emergent Gravity and the Dark Universe" by Erik Verlinde

Youngsub Yoon[1], Ho Seong Hwang[2,3,4]

[1]*Department of Physics and Astronomy, Sejong University, 209 Neungdong-ro Gwangjin-gu, Seoul 05006, Republic of Korea*

[2]*Department of Physics and Astronomy, Seoul National University, Gwanak-gu, Seoul 08826, Republic of Korea*

[3]*SNU Astronomy Research Center, Seoul National University, Seoul 08826, Republic of Korea*

[4]*Australian Astronomical Optics - Macquarie University, 105 Delhi Road, North Ryde, NSW 2113, Australia*

May 1, 2025

Abstract

In "Emergent Gravity and the Dark Universe," Verlinde justifies a formula by three different derivations. However, we discovered that all these three derivations miss the same factor "2/3." If this factor is taken into account, Milgrom's constant used *in* Verlinde's emergent gravity must be given by $cH_0/9 = 0.76 \times 10^{-10}\mathrm{m/s^2}$ instead of Verlinde's original value $cH_0/6 = 1.1 \times 10^{-10}\mathrm{m/s^2}$, which roughly agrees with 1.2×10^{-10} $\mathrm{m/s^2}$, Milgrom's constant used *in* Modified Newtonian Dynamics. However, fitting galaxy rotation curves with Verlinde's emergent gravity shows that the most preferred value for Milgrom's constant in Verlinde's emergent gravity is also $0.76 \times 10^{-10}\mathrm{m/s^2}$, smaller than its most preferred value in MOND. This confirms that the missing factor 2/3 is real.

1 Introduction

Verlinde's emergent gravity tries to answer the missing mass problem without supposing dark matter by suggesting that Newtonian gravity fails in low gravitational accelerations [1]. Like Modified Newtonian Dynamics (MOND), it can explain the Tully-Fisher relation, as Verlinde showed himself. In Modified Newtonian Dynamics, Milgrom's constant a_M, given by $1.2 \times 10^{-10}\text{m/s}^2$ plays a central role. It is the acceleration scale, for which Newtonian gravity begins to fail, and it also appears in the Tully-Fisher relation. What is remarkable about Verlinde's feat is that he obtained the value of a_M in terms of the Hubble constant. According to him, it is given by $cH_0/6$, which is about $1.1 \times 10^{-10}\text{m/s}^2$. Thus, he connected the accelerated expansion of our universe with galaxy rotation curves, which have no obvious relation from the point of view of MOND.

In 2022, Park and us tested Verlinde's emergent gravity by applying it to galaxy rotation curves [2]. We found that Verlinde's emergent gravity explains the galaxy rotation curves well, but a somewhat lower value of a_M than $cH_0/6$ is preferred. Nevertheless, we did not check exactly which value fits the galaxy rotation curves the best.

In 2021, the current first author, found that the factor 2/3 was missing in two independent passages in Verlinde's emergent gravity paper but neglected to publish it. If this factor is taken into account, the value of a_M must be reduced by the same factor, namely

$$a_M = \frac{2}{3}\frac{cH_0}{6} = \frac{cH_0}{9}. \tag{1}$$

In this study, we ran our codes for the galaxy rotation curves again and found out that exactly this value is preferred. This shows that it is indeed plausible that 2/3 factor is missing.

The organization of this paper is as follows. In Section 2, we briefly introduce Verlinde's emergent gravity. In Section 3, we explain the first missing factor of 2/3 in Verlinde's equation. In Section 4, we explain the second missing factor of 2/3 in Verlinde's equation. In Section 5, we derive the corrected value of a_M based on the previous two sections. In Section 6, we present a new argument that the missing factor 2/3 must be present. Those who are not familiar with the details of Verlinde's emergent gravity are encouraged to skip Sections 3, 4, and 5 to start with this section. In Section 7, we show that galaxy rotation curves fit the best with this reduced value of a_M. In Section 8, we conclude our paper.

2 Very brief introduction to Verlinde's emergent gravity

In 2011, Verlinde suggested "entropic gravity" [3]. He argued that gravity is an entropic force. Let us explain what it is. A system always evolves into a configuration that has a higher entropy. Therefore, it tends to move toward a position that has higher entropy. Then, such a movement may apparently seem to be due to a force, but it is actually only due to entropy. This is called the entropic force, a well-known concept in polymer physics.

By assuming the Bekenstein-Hawking entropy for the entropy associated with area, Verlinde successfully derived Newton's law of gravitation and the Einstein field equation. In particular, the fact that the entropy is proportional to area results in Newton's inverse square law. However, it is not possible to prove entropic gravity by experiments, as its theoretical prediction is exactly the same as Newtonian gravity and general relativity.

In 2017, Verlinde went further by suggesting emergent gravity [1]. He assumed that our universe is de Sitter space. De sitter space has an event horizon, thus, also an associated Bekenstein-Hawking entropy. Verlinde assumed that the de Sitter entropy is uniformly distributed in our universe. In other words, if there is a volume, it contains entropy proportional to the volume, due to the uniformly distributed de Sitter entropy. This volume entropy competes with the area entropy introduced in 2011. For a large scale, the volume entropy is relatively important compared to the area entropy, because the volume entropy scales as r^3, while the area entropy only as r^2. Therefore, the gravitational acceleration deviates from the inverse square law, because the volume entropy is no longer negligible.

3 The missing factor 2/3 in the first derivation

In (7.32) of [1], Verlinde tries to show

$$\left(\frac{d-2}{d-1}\right)\int \epsilon'^2_{ij}\, dV = \int_{\mathcal{B}\cap\mathcal{V}_M} dV. \tag{2}$$

Let's calculate the integration on the left-hand side, following Verlinde's logic. Using (7.31), we have

$$\int \epsilon'^2_{ij}\, dV = \int (\nabla_i \nabla_j \chi)^2\, dV = \int (\nabla^2 \chi)^2\, dV. \tag{3}$$

Using (7.25), this is equal to

$$= \int_{\mathcal{B}\cap\mathcal{V}_M} \left(\frac{d-1}{d-2}\right)^2 dV. \tag{4}$$

Plugging this back to the left-hand side of (1), we obtain

$$\left(\frac{d-2}{d-1}\right)\int \epsilon'^2_{ij}\, dV = \left(\frac{d-1}{d-2}\right)\int_{\mathcal{B}\cap\mathcal{V}_M} dV, \tag{5}$$

which is different from (1) by the factor $(d-1)/(d-2)$.

4 The missing factor 2/3 in the second derivation

Let's evaluate the left-hand side of (7.33):

$$\frac{1}{2}\int_{\mathcal{B}\cap\bar{\mathcal{V}}_M} \epsilon_{ij}\sigma_{ij}\, dV = -\frac{1}{2}\int_{\mathcal{B}\cap\mathcal{V}_M} \epsilon_{ij}\sigma_{ij}\, dV = \frac{1}{2}\oint_{\partial(\mathcal{B}\cap\mathcal{V}_M)} u_j\sigma_{ij}\, dA. \tag{6}$$

Using (7.20), we have

$$= \frac{a_0^2}{16\pi G}\oint_{\partial(\mathcal{B}\cap\mathcal{V}_M)} (\epsilon_{kk}\delta_{ij} - \epsilon_{ij})u_j\, dA. \tag{7}$$

where $a_0 = cH_0$. Because ϵ_{ij} is proportional to δ_{ij}, as Verlinde remarks,

$$= \frac{a_0^2}{16\pi G} \oint_{\partial(\mathcal{B}\cap\mathcal{V}_M)} u_i \, dA_i \left(1 - \frac{1}{d-1}\right) = \left(\frac{d-2}{d-1}\right) \frac{a_0^2}{16\pi G} \oint_{\partial\mathcal{B}} u_i \, dA_i. \tag{8}$$

Compared with the right-hand side of (7.33), the above formula has the extra factor $(d-2)/(d-1)$. Thus, the same factor is missing in (7.34) as well.

5 The derivation of Milgrom's constant

In (7.35), Verlinde proceeds as follows:

$$\int_{\partial\mathcal{B}} u_i dA_i = \oint \frac{\Phi_B}{a_0} n_i dA_i \tag{9}$$

This leads to his (7.37), which is

$$\left(\frac{8\pi G}{a_0}\Sigma_D\right)^2 = \left(\frac{d-2}{d-1}\right) \nabla_i \left(\frac{\Phi_B}{a_0} n_i\right) \tag{10}$$

Since the right-hand side must be multiplied by $(d-2)/(d-1)$, from

$$\Sigma_D = -\frac{2g_{Di}}{8\pi G} \tag{11}$$

and $d = 4$, we get

$$g_D^2 = \frac{a_0}{9} \nabla_i (\Phi_B n_i). \tag{12}$$

Thus, we obtain $a_M = a_0/9 = cH_0/9$ instead of $a_M = cH_0/6$.

6 The missing factor 2/3 in the third derivation

Instead of missing factor in Verlinde's mathematical derivations discussed in our earlier sections, Verlinde also obtains Milgrom's constant more heuristically, by a physical argument. In this section, we discuss it and improve it.

As explained in Section 2, de Sitter universe has an entropy associated with volume. If the de Sitter length scale is L, it is given by

$$S_{dS} = \frac{A}{4G\hbar} = \frac{4\pi L^2}{4G\hbar}. \tag{13}$$

As this entropy is uniformly distributed, the entropy the volume $V(r) = (4\pi/3)r^3$ contains is given by

$$S_{DE}(r) = V(r) \frac{S_{dS}}{\frac{4}{3}\pi L^3}. \tag{14}$$

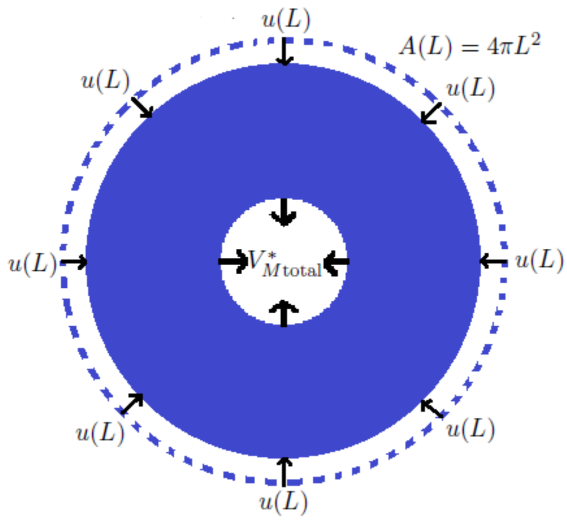

Figure 1: $u(L)$, the displacement at L is interpreted to be due to the removal of $V^*_{M\mathrm{total}}$ at the center.

Now, we explain the area entropy, which competes with the volume entropy. In the presence of mass M at the center $r = 0$, the distance shortens by the general relativistic effect. In turn, it shrinks the area $A(r)$ $\left(= 4\pi r^2\right)$, which leads to a decrease in area entropy. The area entropy decrease due to the presence of mass M at distance r is given by

$$S_M(r) = -\frac{2\pi M r}{\hbar}. \tag{15}$$

$S_M(r)$ competes with $S_{DE}(r)$. If $S_M(r)$ eats up all the volume entropy $S_{DE}(r)$ (i.e., $|S_M(r)| > S_{DE}(r)$), no volume entropy is left, so Newtonian gravity reigns. On the other hand, if $S_M(r)$ cannot remove all of $S_{DE}(r)$, volume entropy is present, which leads to the deviation from Newtonian gravity. Here, we can introduce $V_M(r)$, the volume that would have contained $|S_M(r)|$, by the following relation

$$S_M(r) = -V_M(r)\frac{S_{dS}}{\frac{4}{3}\pi L^3}. \tag{16}$$

Compare this with (14).

Verlinde, then, assumes that $S_M(r)$ removes the space and thus causes elastic displacement. In fact, Sections 3 and 4 are based on the language of elasiticity. First of all, at the horizon $r = L$, if there is a mass M present at $r = 0$, the position of the new horizon is given by

$$1 - \frac{r^2}{L^2} - \frac{2GM}{r} = 0 \tag{17}$$

as the metric component must be zero at the horizon. Then, in the linear order, the horizon position moves from $L \to L + u(L)$, where $u(L) = -2GM$. Verlinde, then, assumes that the displacement $u(L)$ is due to the removal of volume at the center. See Fig. 1. To derive the right value of $u(L)$, the removed volume at the center $V^*_{M_{\text{total}}}$ must satisfy the following equation.

$$u(L) = -\frac{V^*_{M_{\text{total}}}}{A(L)}. \tag{18}$$

If we calculate the value of $V^*_{M_{\text{total}}}$, we obtain

$$V^*_{M_{\text{total}}} = \frac{3}{2} V_M(L). \tag{19}$$

(Recall the definition of $V_M(r)$ in (16).) Then, for $u(r)$, the displacement u at position r, Verlinde assumes that it is also due to the removal of volume. If we denote the removed volume by $V^*_M(r)$, it satisfies

$$u(r) = -\frac{V^*_M(r)}{A(r)}. \tag{20}$$

He also assumes that the removed volume at the center is proportional to the entropy it would have contained. Thus, we have

$$V^*_M(r) \propto S_M(r). \tag{21}$$

Combining with $S_M(r) \propto V_M(r)$, and (19), Verlinde obtains

$$V^*_M(r) = \frac{3}{2} V_M(r). \tag{22}$$

However, instead of using such an assumption, it would be more natural to assume that $V_M(r)$, the volume that would have contained the removed entropy, is removed at the center, instead of $V^*_M(r)$. Then, $u(r)$ in (20) will be reduced by factor $2/3$ as in (8). Then, Milgrom's constant will be reduced by the same factor as shown in Section 5.

7 Comparison with galaxy rotation curves

In [2], Park and us fitted galaxy rotation curves with Verlinde's emergent gravity. We assumed $a_M = a_0/6$ and considered two values of a_0: de Sitter value, which is $a_0 = cH_0 = 6.7 \times 10^{-10}\text{m/s}^2$ and quasi de Sitter value, $a_0 = 5.4 \times 10^{-10}\text{m/s}^2$, which was considered in [4]. If $a_M = a_0/6$ is used, our value $a_M = cH_0/9$ would imply $a_0 = 4.5 \times 10^{-10}\text{m/s}^2$. Indeed, in [2], we found that the quasi de Sitter value, which is closer to our value of a_0 than the de Sitter value, fits galaxy rotation curves better. We repeated our earlier analysis by changing a_0. See Table 1. We see that $a_0 = 4.55 \times 10^{-10}\text{m/s}^2$ fits the best. In this case, we have $a_0/6 = 0.76 \times 10^{-10}\text{m/s}^2$, which coincides with $cH_0/9 = 0.76 \times 10^{-10}\text{m/s}^2$.

a_0 (10^{-10}m/s^2)	μ	$\mu_{\rm err}$	σ	$\sigma_{\rm err}$
4.7	-0.005	0.002	0.130	0.002
4.6	-0.001	0.003	0.128	0.003
4.55	0.000	0.003	0.128	0.003
4.5	0.002	0.003	0.128	0.003
4.4	0.005	0.003	0.128	0.003

Table 1: Galaxy rotation curve fit for various a_0. μ and σ denotes the mean and the standard deviation of $\log_{10}(g_{\rm obs}/g_{\rm Ver})$ where $g_{\rm obs}$ is the observed gravitational acceleration and $g_{\rm Ver}$ is the Verlinde predicted gravitational acceleration. We see that $a_0 = 4.55 \times 10^{-10}$m/s^2 fits the best.

8 Conclusion

In this paper, we showed in three independent methods that the factor 2/3 is missing in Verlinde's formula. The first and second methods were due to apparent calculation errors, and the third method was more subtle, involving assumptions in theory. However, the first and second calculation errors are sufficient to theoretically prove that there should be an additional factor of 2/3, regardless of the subtlety of the assumption regarding the third method. Moreover, it has already been observationally proven that factor 2/3 must be present there. Therefore, it is settled.

Acknowledgement

This work was supported by the National Research Foundation of Korea [NRF-2021R1A2C1094577(HSH), NRF-2022R1A2C1092306(YY)]. H.S.H also acknowledges the support of Samsung Electronic Co., Ltd. (Project Number IO220811-01945-01), and Hyunsong Educational & Cultural Foundation.

References

[1] E. P. Verlinde, "Emergent Gravity and the Dark Universe," SciPost Phys. **2**, no.3, 016 (2017) doi:10.21468/SciPostPhys.2.3.016 [arXiv:1611.02269 [hep-th]].

[2] Y. Yoon, J. C. Park and H. Seong Hwang, "Understanding galaxy rotation curves with Verlinde's emergent gravity," Class. Quant. Grav. **40**, no.2, 02LT01 (2023) doi:10.1088/1361-6382/acaae6 [arXiv:2206.11685 [gr-qc]].

[3] E. P. Verlinde, "On the Origin of Gravity and the Laws of Newton," JHEP **04**, 029 (2011) doi:10.1007/JHEP04(2011)029 [arXiv:1001.0785 [hep-th]].

[4] A. Diez-Tejedor, A. X. Gonzalez-Morales and G. Niz, “Verlinde's emergent gravity versus MOND and the case of dwarf spheroidals,” Mon. Not. Roy. Astron. Soc. **477**, no.1, 1285-1295 (2018) doi:10.1093/mnras/sty649 [arXiv:1612.06282 [astro-ph.CO]].