Response to the Referee 2

Strengths

-interesting SU(2)-symmetric setup to study operator entanglement

-accurate numerical data

We thank the Referee for the positive assessment of our SU(2)-symmetric dissipative quantum many-body model for studying the operator entanglement growth and the numerical simulation we performed.

Weaknesses

no physical insight for the prefactors of the operator entanglement growth

We thank the Referee for raising this point. However, we don't think the lack of analytical interpretation of the prefactors would reduce the quality of our manuscript and affect the corresponding fulfillment of the SciPost Physics acceptance criterion, as we clarified below.

Report

The author investigates the operator entanglement entropy in a dissipative system in the presence of SU(2) symmetry, considering dissipation beyond dephasing. This work makes a valuable contribution to the study of open quantum systems by extending previous findings on U(1)-symmetric systems to more complex symmetry structures and dissipative mechanisms. The results provide valuable insights into the interplay between symmetry, dissipation, and entanglement dynamics, which are of substantial interest in the field of quantum many-body physics and classical simulability of quantum systems. The analysis is rigorously performed and the results are clearly presented. However, the paper is lacking an interpretation of the logarithmic growth of the operator entanglement, as also stressed in my comments below. For this reason I believe that the paper is more suitable for Scipost Physics Core.

We thank the Referee for the careful reading of our manuscript, especially for the positive assessment that "*This work makes a valuable contribution to the study of open quantum systems.... The results provide valuable insights into the interplay between symmetry, dissipation, and entanglement dynamics.... The analysis is rigorously performed and the results are clearly presented.*" From the comments raised by the Referee, we understand that the Referee mainly concerns with the lack of analytical interpretation for the prefactors of the operator entanglement growth. However, we would like to mention that in the U(1)-symmetric open quantum many-body system with dephasing considered in [Wellnitz *et al.*, PRL 129, 170401 (2022)], the analytical interpretation for the prefactor of late-time logarithmical growth of operator entanglement is possible since the late-time operator entanglement in this case can be fully captured by the classical Shannon entropy associated with the probabilities for the half system being in different U(1) sectors, which is further related to the well-known classical symmetric simple exclusion process by the well-established perturbation theory in the strong dephasing limit, thus significantly simplifying the task of interpretation. In our SU(2)-symmetric case, both the classical Shannon entropy of probabilities and the symmetry-resolved operator entanglement contribute nontrivially to the late-time operator entanglement, which makes the interpretation extremely hard. Moreover, since our model contains dissipation proportional to the dipole interaction between neighbor sites, even in the strong dissipation limit it is still hard to identify the density matrix ρ satisfying $\mathcal{L}_0[\rho] \equiv \sum_i (L_i \rho L_i^\dagger - \{L_i^\dagger L_i, \rho\}/2) = 0$ and the corresponding projector $\mathcal{P}$ onto these states, which makes the usual perturbation theory also unrealistic for our model. For this, we do not have an analytical interpretation for the prefactors.

Nevertheless, we don't think the lack of analytical interpretation of the prefactors would reduce the quality of our manuscript, which indeed fulfills the SciPost Physics acceptance criterion. On the one hand, our work established a synergetic link between the quantum information theory, open quantum many-body dynamics, and non-Abelian symmetries. While the operator entanglement, an important concept in quantum information theory to analyze the entangling capabilities of quantum evolution, has been used to study the closed nonequilibrium quantum many-body physics like quantum chaos and information scrambling very recently (see Refs. [53-55] of our manuscript), the application to the open quantum many-body systems is quite rare, especially its interplay with the symmetries. Although the Abelian U(1)-symmetric case with dephasing has been considered in [Wellnitz *et al.*, PRL 129, 170401 (2022)], it was our work that investigated for the first time the influence of the much more complicated non-Abelian symmetry on the operator entanglement dynamics of open quantum many-body systems and obtained interesting new results. Therefore, our work indeed satisfies the SciPost Physics acceptance criterion that "Provide a novel and synergetic link between different research areas." On the other hand, our manuscript also "Detail a groundbreaking theoretical/experimental/computational discovery." In [Wellnitz *et al.*, PRL 129, 170401 (2022)], the discussions were limited only to the U(1) symmetry with dephasing and no explicit conjecture about other situations was provided, from which it was not known whether the late-time logarithmic growth behavior of operator entanglement can be applied or not to other symmetries and/or dissipations. It was our work that provided for the first time a positive evidence that the logarithmic growth of operator entanglement at long times is a generic behavior of dissipative quantum many-body dynamics with U(1) as the symmetry or subsymmetry (like the SU(2)-symmetric case) and for more broad dissipations beyond dephasing, which is absolutely a groundbreaking discovery as now the logarithmic growth behavior of operator entanglement can be applied to much more broad dissipative quantum many-body systems than the one considered in [Wellnitz *et al.*, PRL 129, 170401 (2022)].

With the above points, we would like to reemphasize that our work indeed fulfills the SciPost Physics acceptance criterion despite the lack of analytical interpretation for the prefactors and is suitable for publication in SciPost Physics. We hope the Referee will also agree with this.

Suggestions:

a) The claim of non universality of the prefactor η of the logarithmic growth in Fig. 1 appears not completely justified. In fact it could be that the putative non universality can be attributed to the finite times that can be reached with the numerics. I would suggest to rephrase the discussion leaving the scenario of universality open.

We thank the Referee for the suggestion. The claimed nonuniversality of prefactor η in our manuscript comes from the numerical tendency exhibited in Fig. 1(d). We agree with the Referee that without analytical insights, we cannot fully rule out the possibility of universality in the time limit $t \to \infty$. For this, in this resubmission we have rephrased the discussion as

"We note that unlike the U(1)-symmetric case with dephasing [38], from the numerical tendency exhibited in Fig. 1(d) both the prefactor η and offset S_0 seem to be nonuniversal and depend on the value of the dissipation strength. However, it should be mentioned that whether η converges universally to a fixed value like Ref. [38] or not in the limit $t \to \infty$ is still an open question due to the limited time achieved in our numerical simulation."

b) In Fig. 2 it is not clear what is the prefactor of the log growth. Is it the same for both the number entropy and the entanglement entropy?

We thank the Referee for raising this point. In this resubmission, we have added the prefactors of the number entropy $-\sum_S p_S \log_2 p_S$ and entanglement entropy $\sum_S p_S S_{\mathrm{op},S}$ as inset plots of Figs. 2(c) and 2(d), respectively. From the numerical results, they are different from each other.

c) From fig 3 it is not clear what would be the growth of the number entropy? Also, what is the prefactor of the log growth? Similarly, in Fig.4 there is no discussion of the prefactor of the log growth.

We thank the Referee for raising this point. As we clarified in the main text of this resubmission, the number entropy in the magnetization sectors is given by $-\sum_{S_z} p_{S_z} \log_2 p_{S_z} \simeq \log_2 \delta + \log_2 \sqrt{2\pi e}$. The corresponding growth is governed by the time dependence of δ, which follows the power law $\sim (tJ)^\alpha$ at late times as we show in Fig. 3(b). Therefore, the number entropy follows a logarithmical growth law with prefactor α at late times. In this resubmission, we have improved the discussions as

"Obviously, the classical Shannon entropy of the probabilities in magnetization sectors, $-\sum_{S_z} p_{S_z} \log_2 p_{S_z} \simeq \log_2 \delta + \log_2 \sqrt{2\pi e}$, which follows a logarithmical growth law with prefactor α, also cannot capture the whole prefactor η of the logarithmic growth of operator entanglement at late times; cf. Fig. 1(d)."

and added the prefactors of Fig. 3(d) in the insert. We also added the prefactors of Figs. 4(a) and 4(b) in the caption:

"The black dashed lines indicate the logarithmic growth of operator entanglement at late times (log-scale time axis), with the prefactor $\eta \approx 0.24$ for (a) and ≈ 0.60 for (b), respectively."

Recommendation

Accept in alternative Journal (see Report)

We thank again the Referee for the review of our manuscript and the helpful comments/suggestions. Our manuscript has been carefully updated according to the above response. With all of the comments and suggestions being addressed, we wish the Referee would now agree the publication of our manuscript in SciPost Physics.