
Report of Referee 3

Strengths

1. The manuscript gives an important physical result and uses appropriate evidence to justify the claim.

2. The calculations are thorough and the results are convincing.

3. This paper proves that there is an extraordinary transition in the tricritical O(N) models in 3d, which is a significant result that can potentially be observed in experiments and/or numerical simulations.

We sincerely appreciate the referee's positive evaluation of our work.

Weaknesses

1. The presentation of the paper could be improved, and the main result can be emphasized more (see report).

2. There are many grammatical errors and typos in the manuscript as it stands, which need to be corrected.

We thank the referee for the constructive feedback. Following the recommendations in the report, we have revised the manuscript to improve its presentation and better emphasize the main result. Additionally, we have carefully proofread the text to correct grammatical errors and typos.

Report

The presented manuscript studies the extraordinary transition for the tricritical O(N) model using RG. The authors calculate the correlators (and layer susceptibilities) upto one loop order in the $d = 3 - \epsilon$ expansion (the upper critical dimension is 3). Using the correlators, the authors are able to extract various CFT data including the boundary OPE coefficients for the one-point functions of displacement and tilt. The authors then go on to show using an RG argument that the epsilon expansion results imply that the tricritical $O(N)$ model has an extraordinary transition where the bulk undergoes a phase transition in the presence of an ordered surface. This is interesting and relevant because the Mermin-Wagner theorem prevents the spontaneous symmetry breaking of continuous symmetries in 2d, but we can see that the $O(N)$ symmetry can be spontaneously broken on the 2d boundary of a 3d system. The methods used in the work are traditional, and well established. It is justified to use the epsilon expansion here because unlike the regular O(N) models the upper critical dimension here is 3, not 4.

We thank the referee for the thoughtful summary of our work and for recognizing the significance and relevance of our results.

My only minor qualm is with the structure of the paper, which I leave for the authors to ponder. I think the reading experience and flow will be enhanced if section 3 is relegated to an appendix. The calculations are essential for what comes after, yes, but it also makes for long equations spanning half a page. The physical results are already emphasized in section 1.1. The technical section 3 might deter readers from appreciating the important discussions in sections 4 and 5.

We thank the referee for the suggestion regarding the paper's structure. While we have maintained the overall organization, we have moved the detailed derivations and lengthy equations from Section 3 to the appendix, retaining only the essential discussions in the main text. Additionally, we have added a note at the beginning of Section 3 to guide readers who may wish to skip the technical details of the loop calculation.

Finally, the grammar of the paper needs improving. I found many errors and typos while reading, some of which I point out in the list of changes. Please attend to them and proofread the manuscript once again. After the grammatical errors are ironed out, I think this paper would be very suitable for publishing in SciPost Phys. The content is original and interesting, and the derivations support the claim.

We thank the referee for the careful reading of our manuscript. We have thoroughly proofread the text to correct grammatical errors and typographical mistakes, including those noted in the referee's suggested changes, to the best of our ability.

Requested changes

1. Abstract and elsewhere - "2D dimensions" should be replaced by either "2d" or "two-dimensions".

We thank the referee for the suggestion. We have corrected all "2D dimensions" in the manuscript with "2d" or "two dimensions".

2. In the first paragraph of introduction, the word "enhance" reads weird, as in "the presence of a boundary enhances the bulk CFT". Perhaps it should say imbibes additional structure.

We thank the referee for the suggestion. We have revised the sentence to: "The presence of a boundary enriches the bulk CFT by introducing additional conformal data".

3. Eq. (1), c_Δ not defined. I suggest moving the line below Eq. (3) higher, below Eq. (2).

We thank the referee for the suggestion. We have moved this part to section 2 following the advice of the other referee. And we have added the explanation of c_Δ just below the corresponding equation.

4. Eq. (4), p is not defined around this equation, although clear from context that it is the momentum in directions parallel to the boundary.

We thank the referee for the suggestion. We have moved this part to section 2 following the advice of the other referee. And we have added a brief explanation for p below the corresponding equation, which is also shown as follows: "Equivalently, this corresponds to the zero-momentum ($p = 0$) component of the connected two-point function in Fourier space, where p denotes the momentum in directions parallel to the boundary."

5. In Section 5, the line above Eq. (93), "couple the ordinary transition action to the nonlinear sigma model according to $O(N)$ symmetry" should be "restoring/respecting $O(N)$ symmetry"

We thank the referee for the suggestion. We have modified the sentence to: "respecting $O(N)$ symmetry".

6. In the paragraph above Eq. (95), the authors state "The normal boundary condition is actually equivalent to the ordinary transition". This is not correct, at least for the $O(N)$ models. In this case, after doing the entire RG analysis, we realize that the boundary is ordered. But a priori these are different boundary conditions and should be treated as such. Extraordinary transition is the spontaneous symmetry breaking of $O(N)$ symmetry whereas normal boundary condition explicitly breaks $O(N)$ symmetry by an external field.

We thank the referee for the comment. In the revised manuscript, we use the terms "normal fixed point" and "extraordinary fixed point" to refer to the IR fixed point of the BCFT corresponding to the normal boundary condition and the extraordinary boundary condition, respectively.