Reply to Referee 2

This paper investigates how noise affects random quantum circuits in quantum computing. The authors focus on two types of random circuits:

- *The original circuit, composed of layers of random two-qubit gates followed by a random permutation of qubits. These circuits are used in quantum volume benchmarking.*
- *The solvable circuit, composed of layers of random two-qubit gates, followed by a global Haar unitary, a random permutation, and another global Haar unitary.*

They study the impact of two main sources of noise on these circuits: faulty two-qubit gates and faulty permutations. To quantify the effects of these errors, the paper analyzes the entanglement fidelity between the target and noisy circuits, averaged over all random instances of these circuits. The authors derive an analytical expression for the fidelity decay in the solvable model, showing its dependence on the number of qubits, the number of circuit layers, and the noise parameters. The results show that even for small systems, the target and noisy circuits can be quite different. Numerical simulations indicate that the analytical predictions for the solvable model closely match the numerical results for the better-motivated original model, suggesting that both circuits exhibit similar fidelity decay trends under noise.
Overall, I believe this study was well-conducted and represents a solid piece of research that deserves publication in SciPost Physics, primarily due to its technical contributions.

We thank the referee for their careful reading and the positive evaluation of our work.

1.-The motivation and relevance of this work were not entirely clear to me. While I understand that these circuits are related to quantum volume, this is just one possible noise model for quantum volume circuits, and quantum volume itself is only one of several benchmarking methods for quantum computers. Are there any other possible applications?

We are grateful to the referee for raising this concern. The random circuit studied in our work is the one proposed in the original definition of Quantum Volume [2]. However, we would like to emphasize that random quantum circuits are not only used in randomized benchmarking in quantum computing. They are of great interest in simulating generic random dynamics and understanding quantum chaos, quantum complexity and thermalization among others. We have expanded the paragraph in the introduction, where we discuss the circuit with random permutations emphasizing the versatility of RQCs, hopefully answering the referee's concern about other possible applications:

"For example, the brick-wall circuit—consisting of a sequential alternation of leftward and rightward single-qubit shifts—is a paradigmatic model of local random quantum circuits (RQCs) [7]. It has been studied extensively in the context of information spreading [9], thermalization [12], and measurement-induced phase transitions [8]. In contrast, random permutations have been used to model black hole dynamics with non-local interactions [6, 11], to establish bounds on entanglement generation [3, 10], to study pseudo-randomness and unitary k-designs—ensembles that reproduce Haar-random statistics up to the k-th moment [4, 5]—and to investigate quantum complexity [1], among other applications."

We agree with the referee in that, although widely implemented, Quantum Volume is not the only figure of merit. However, as mentioned in the manuscript, it is a versatile, architecture-agnostic, difficult-to-cheat benchmark, with clear, meaningful interpretation. Thus we consider it worth investigating.

We address the concerns about the chosen error model while answering question number 3. Finally, we have added a sentence to the final paragraph of the conclusions highlighting a possible research direction beyond the quantum-computing-inspired topics already discussed.

"In addition, it could be interesting to explore the connection between average fidelity in the generic models considered and out-of-time order correlators (OTOCs), inspired by the known relation between the Loschmidt echo and OTOCs in systems governed by time-independent Hamiltonians [13]. "

2.- How does this work relate to previous research? Have similar studies been conducted before?

We thank the referee for raising this concern, which overlaps with the second major issue of Referee I. We tried to thoughtfully address this concern in the answer to I Referee's question.

3.- Why did you choose this error model? Why did you choose this particular form for the solvable model? The global Haar random unitaries seem to be included purely for convenience. Could a simpler circuit yield the same predictions?

We thank the referee for raising these questions. We consider noise in the unitary gates, and noise in the permutations, each of them reflecting the nature of the considered gate, as it is discussed in detail in section 2.1. We extended the last paragraph of this section to clarify possible issues as cited in the answer to the second major concern of Referee I.

The solvable model was indeed introduced to enable analytical study. It is important to note that computing the average of the fidelity Eq. (1) for the original model is a very hard task. The reason is that the random permutations induce uncontrolled correlations between layers that preclude to computation of the average as a product of averages. On the other hand large unitary, after averaging, "decouple" permutations from 2-qubit gates mitigating this problem. The introduction of large random unitaries, fortunately, does not substantially modify circuit behavior, since on the level of 2-design – calculation of average fidelity their action is akin to the original quantum circuit. To clarify this problem we have modified the first paragraph of Section 3. We also addressed the issue of similarity between the original and solvable model in the answer to a first major concern of Referee I.

Finally, to the best of our knowledge, we do not envisage a simpler circuit that allows us to obtain an analytical expression.

4.- I find the text around equation (B13) unclear.

We have modified the text accordingly by adding a new paragraph which clarifies this issue:

"It is convenient to exploit the fact that the GUE measure is invariant under unitary transformations. In particular, this implies that the eigenvectors of H do not favour any specific direction in Hilbert space. Therefore, the unitary matrix U that diagonalizes $e^{i\alpha H}$, namely

$$e^{i\alpha H} = U D_\lambda U^\dagger, \quad \text{with} \quad D_\lambda = \text{diag}(e^{i\alpha\lambda_1}, \ldots, e^{i\alpha\lambda_d}),$$

must be distributed according to the Haar measure. As a result, the average in Eq. (B.12) can be decomposed into an average over two copies of the Haar-distributed unitaries, as discussed above, together with an average over the eigenvalues encoded in $D_\lambda \otimes D_\lambda^*$, which must be performed with respect to the GUE joint probability distribution. "

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