Reply to Referee 1

Marco Cirelli, Arpan Kar

We thank Referee 1 for carefully going through the manuscript and providing us with his/her in-depth comments. Please find below our responses addressing his/her comments in the order they were presented in the report. During the re-submission process, we are going to upload a modified version of the manuscript that we have prepared highlighting the changes incorporated to address the referee's comments.

Response to major comments:

1. In order to address the issue regarding the systematic uncertainty in the astrophysical background model, we have added in the modified version of the draft a new paragraph. In this paragraph we discuss in text the possible effects of such uncertainty on our results.

   In order to illustrate this issue more thoroughly for the referee, we also provide here below Fig. 1. The figure shows the maximum level of relaxation in our projected sensitivities. As discussed in the manuscript, we re-estimate our previous projected sensitivities (corresponding to the Amego telescope) by enhancing each of the fiducial photon backgrounds up to a constant factor of 2 (independent of the photon energy). In Fig. 1, the dotted and dashed blue lines indicate the projections obtained previously (considering the SNR and Fisher approaches, respectively), while the associated blue band in each case indicates the range of variation in the projection if one modifies the astrophysical backgrounds in the above-mentioned way. As can be seen from this figure, the inclusion of such systematic uncertainties in the background modeling has a relatively mild effect on our results. In order to avoid too many figures in the paper (which already features a lot of different plots), we prefer not to include this present one in our main draft, and we limit to the text discussion.

2. In order to address the issue of the propagation of electrons/positrons properly, we have now added in our 'Results and discussion' section a new extended sub-section (replacing appendix A.4). In this new sub-section we discuss the effects of propagation of electrons/positrons on our results in detail.

   We provide here below Fig. 2 that we have prepared for the modified paper showing the level of changes in our bounds and projected sensitivities if the effects of full propagation of $e^{\pm}$ in the Galaxy is included. We have considered two propagation models, named here as 'prop. 1' and 'prop. 2', which have been used in the past for the study related to the secondary photon emissions towards the GC.

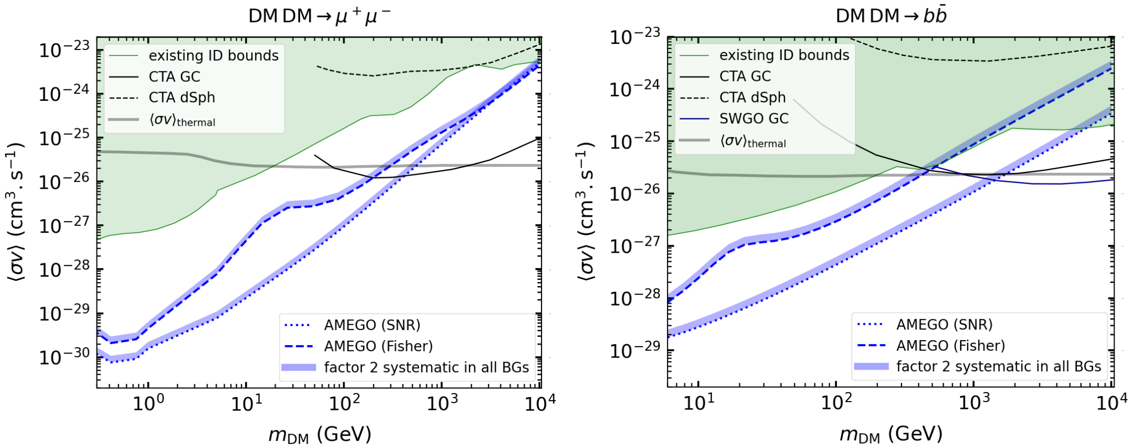

Figure 1: *An illustration for the maximum level of relaxation in our projected sensitivities due to the inclusion of systematic uncertainties in the background modeling. The left and right panels refer to $\mu^+\mu^-$ and $b\bar{b}$ annihilation channels, respectively. The dotted and dashed blue lines indicate the projections for* AMEGO *obtained previously considering the SNR and Fisher approaches, respectively. They are the same as the ones shown in Fig. 3 of the present version of the paper. The associated blue band in each case indicates the range of variation in the projection, if each component of the fiducial photon background is conservatively enhanced up to a factor of 2 (independent of the photon energy).*

Model 'prop. 1' refers to the "model z04LMS" from [arXiv:1008.4330] which was adopted in [arXiv:1101.1381] to estimate the different Galactic photon backgrounds considered in our work. Model 'prop. 2' refers to the "model A" adopted in [arXiv:1409.0042] to estimate the Galactic background fluxes. This model was used in [arXiv:2411.00087] to rescale the Galactic backgrounds to the region 10° cone around the GC which is our target ROI. Notice that Model 'prop. 2' is actually very similar to 'prop. 1' for relativistic $e^{\pm}$; the only difference is that 'prop. 2' includes a convective wind which 'prop. 1' does not.

From Fig. 2 here one can see that including a detailed and involved numerical simulation of the propagation of $e^{\pm}$ in the Galaxy under some commonly used propagation models (that are used to estimate the Galactic photon backgrounds towards the GC) the bounds and the projections of MeV telescopes on the weak-scale DM parameter space can be weakened maximally by a factor $\sim 2-3$ depending on the DM mass. Hence, this discussion shows that the conclusions based on the results obtained adopting the semi-analytic approach to estimate the Galactic distribution of DM induced $e^{\pm}$ remain the same.

Response to minor comments:

We have now divided Figs. 3 and 5 from the present version of the paper into more

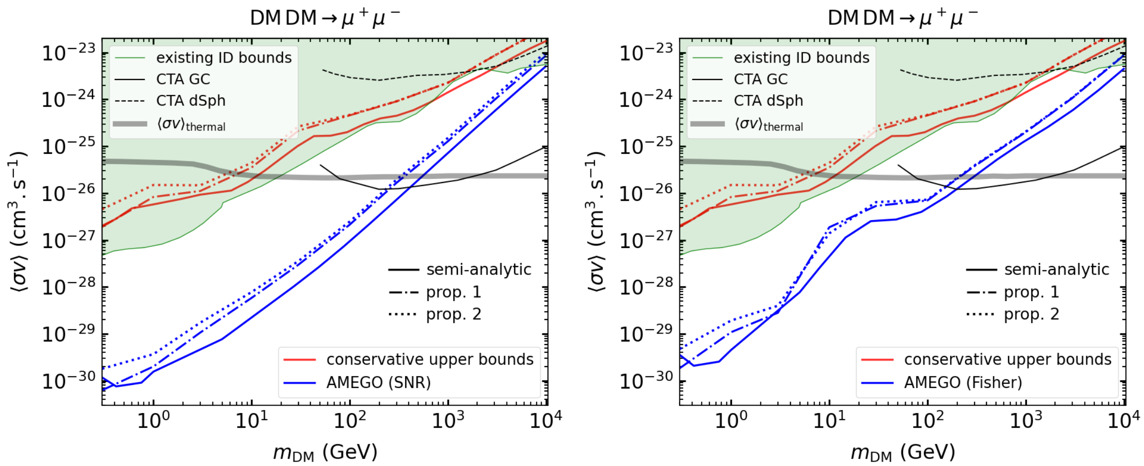

Figure 2: *Effects of the propagation of* $e^{\pm}$ *on the estimated bounds and projections are shown for the* $\mu^+\mu^-$ *annihilation channel. The projections are shown for* AMEGO *and are obtained using the two statistical approaches, SNR (left panel) and Fisher (right panel). The red and blue solid curves are respectively the same bounds and projections shown in fig. 3 of the paper and correspond to the case where the* $e^{\pm}$ *distribution in the Galaxy is obtained using the semi-analytic method (described by Eq. 5 of the paper). The red and blue broken curves, on the other hand, show the results obtained with the full propagation of* $e^{\pm}$ *in the Galaxy, for the two models discussed in the text.*

figures, as the referee suggested. Figure 3 is now divided into two Figures. Figure 5 has now divided into three separate sub-figures. We have also modified the text accordingly.

We hope that with these modifications the manuscript will now be suitable for publication.

Best regards,
the authors.