Reply to Referees' comments

Freelance Holography, Part I: Setting Boundary Conditions Free in Gauge/Gravity Correspondence

We would like to thank the referee for carefully reading the paper and for constructive comments. We have tried to address them and improve our paper accordingly. Please see below for more details. For a better visibility, we have kept the changes in the paper in red.

Reply to Referee 1

We thank the referee for the comments.

-since one of the nice things about the covariant phase space formalism is that it provides a way to define conserved charges and their algebras, it would be good if the authors would comment on how these quantities depend on the ambiguities discussed

That's true, the Covariant Phase Space Formalism (CPSF) is indeed a powerful framework for analyzing surface charges in both gauge and gravitational theories. There exists a substantial body of literature on surface charges and their behavior under various ambiguities [1, 2, 3, 4, 5, 6, 7, 8]. However, our main objective in our Freelance Holography Program has been to study and analyze implications of the powerful CPSF in holography. Our results demonstrate that CPSF has broader applications beyond its conventional use in computing surface charges, asymptotic symmetries, and their algebra. In particular, our findings suggest that CPSF offers a natural and fruitful foundation for constructing holographic dualities beyond the standard Dirichlet boundary conditions at conformal/causal boundary of AdS. The study of conserved charges lies beyond the scope of our present program, as this question has already been investigated.

-in previous works like Setting the boundary free in AdS/CFT, authors would find counterterms that lead to finite quantities in the bulk for non-Dirichlet boundary conditions. In this paper, counterterms are considered for Dirichlet boundary conditions and then the boundary conditions are changed. It would be good if the authors could comment on the relation between the two.

Thank you for pointing this out. Indeed, in [9], the authors define Neumann and mixed boundary conditions in terms of the renormalized Brown–York energy-momentum tensor (rBY-EMT) at infinity. In this sense, our Neumann boundary condition is consistent with theirs. We have added a comment at the end of subsection 5.2 to acknowledge and cite their work.

However, the main point we would like to emphasize is that the phase space of a theory should be defined in terms of physical, renormalized variables. Constructing a phase space using non-renormalized quantities, such as the extrinsic curvature in gravity, is not physically meaningful. Our formalism is specifically designed to work with renormalized variables as the fundamental labels of the phase space. As a result, various types of boundary conditions, including Dirichlet, Neumann, conformal, conformal conjugate, and others, are naturally formulated within this framework.

-the analysis in this paper is in the saddle point approximation at large N, but the authors conjecture that can be extended beyond this approximation. Do the authors have in mind any ways to test their proposal?

That's a very good question. As an illustrative example, consider general relativity in three spacetime dimensions with a negative cosmological constant. In this case, the double-trace $\mathrm{T\bar{T}}$ deformation can be interpreted as a mixed boundary condition even at finite N [10]. This example suggests that the interpretation of multitrace deformations as modifications of boundary conditions may remain valid beyond the strict large-N limit.

As an ongoing project, we are currently studying $3d$ general relativity with various types of boundary conditions, aiming to establish how they correspond to multitrace deformations at finite N. We have added a paragraph at the end of the discussion section to highlight your insightful question.

-regarding the discussion in section 6.2 about non-marginal deformations: the analysis in the paper is focused on AdS/CFT. In the case when we first deform the boundary theory such that it is no longer a CFT, do the authors suggest that the analysis still holds? It would be interesting to add a comment about validity outside AdS/CFT.

Let us consider equation (2.6a), which presents the standard formulation of the AdS/CFT correspondence. While the first term in the boundary action, S_{CFT}, describes a conformal field theory, the presence of the single-trace deformation term $\mathcal{O}_i\mathcal{J}^i$ modifies the original theory. As a result, the full boundary action no longer defines a CFT. This reflects the standard Wilsonian perspective on deformation: the introduction of non-marginal operators triggers a renormalization group (RG) flow away from conformality. In the gravity side, these correspond to asymptotically AdS spaces, i.e. deformations of the AdS space with the same asymptotic/conformal/causal boundary as AdS.

Furthermore, the construction of holography at finite distance, which is the central focus of Freelance Holography II, requires introducing an irrelevant double-trace $\mathrm{T\bar{T}}$ deformation to the asymptotic boundary action. As discussed in Part II, this leads to a framework more accurately described as C-AdS/QFT correspondence, where C-AdS is AdS space cutoff at a finite r.

One might be concerned that the inclusion of *irrelevant multitrace* deformations (with scaling dimension $\Delta \ll N$) could undermine the validity of a large-N analysis, as such deformations typically become significant at high energies due to their irrelevant nature. To address this, we emphasize that the large-N limit is taken first, and in this regime, the multitrace structure of the deformation suppresses its effect. In other words, for irrelevant multitrace deformations, two competing effects are at play: their growth at high energies and their suppression by large-N scaling. In our analysis, we assume a regime in which the latter dominates, and the multitrace character of the deformation ensures its subleading contribution. We have added a comment at the end of section 6 to indicate this point.

Reply to Referee 2

We thank the referee for comments and suggestions. We address the comments/questions as follows.

Weaknesses (1): Few new results; many parts are review of existing literature.

Here, we would like to outline our new analysis and results to clarify this issue:

- First, we provide a new perspective on holography and role and significance of boundary conditions in it. The Covariant Phase Space Formalism (CPSF) is a well-established framework for studying surface charges, asymptotic symmetries, charge algebras, and related structures. In this paper, we have employed CPSF to extend the scope of holography in several directions. This approach opens new avenues for exploring holographic problems through the lens of CPSF.
- Section 3 explores generalized boundary conditions in the AdS/CFT correspondence and *systematically derives* Witten's proposal for gravitational boundary conditions in the presence of multi-trace deformations of the boundary theory, using the CPSF.
- As detailed in the manuscript, the CPSF involves certain inherent ambiguities or freedoms. In this paper, we discuss and uncover the role and physical significance of these freedoms within an extension of holographic framework beyond the standard AdS/CFT, which is specified by Dirichlet boundary conditions at the asymptotic causal boundary of AdS.
- In parts of Section 5, we introduced a new class of boundary conditions that are constructed to yield finite quantities at infinity. It is important to note that the counterterms required for the well-known conformal boundary condition, as defined in terms of the extrinsic curvature, are not fully understood. However, among other interesting boundary conditions, we propose a new type of conformal boundary condition that ensures finiteness. This opens a new avenue for exploring black hole thermodynamics in novel ensembles, as well as for studying linearized gravity within this new family of boundary conditions, which are directions we intend to pursue in future work.
- Section 6 introduces a new class of hydrodynamic deformations, which generally include multi-trace deformations defined by functions of the trace of the renormalized Brown–York energy-momentum tensor.

We have added a new paragraph to the end of introduction to highlight these new results and viewpoints.

(1) The authors should highlight throughout the manuscript the specific new results, beyond stating that their work extends the analysis of [33].

In [33], the authors study holography with Dirichlet, Neumann, and mixed boundary conditions, and argue that for non-Dirichlet boundary conditions, the boundary CFT becomes coupled to boundary gravity. They construct an explicit model of a boundary theory consisting of a CFT coupled to gravity and explore some of its implications. In contrast, our framework is significantly more general. Among other new results summarized under Weakness (1), it applies to arbitrary boundary conditions and to any gravitational theory in the bulk. The specific boundary conditions considered in [33] appear merely as special cases within our broader construction. As such, the only overlap between our work and theirs lies in the treatment of holography with Neumann boundary conditions. We have added comments at the end of subsection 5.2 and the discussion section to acknowledge and clarify this point.

(2) A relevant article analyzing (finite) conformal boundaries in asymptotically AdS4 the authors should consider (and cite along with Refs. [87] and [88]) is [hep-th/2412.16305].

Thank you for bringing it to our attention. We have added the reference accordingly.

(3) In Section 5.3, the authors consider boundary conditions that differ from standard conformal boundary conditions (CBCs). Standard CBCs are those are those which fix both the conformal class of the induced boundary metric and the trace of the extrinsic curvature. Instead, the authors characterize their CBCs are fixing the conformal class of the induced boundary metric and the trace of the renormalized Brown-York stress-tensor, their reasoning being that this will ensure finiteness of the on-shell action. I have two points of confusion here that I would like clarified.
(i) First, it seems the authors rely on working with a Gibbons-Hawking-York boundary term suitable for making the variational problem well-posed for Dirichlet boundaries (see Eq. (5.1)). When CBCs are imposed, however, it is known that the necessary Gibbons-Hawking-York boundary term differs by an overall constant factor. The authors should explain why they use the Gibbons-Hawking-York term appropriate for Dirichlet boundaries here, even though they impose different CBCs.
(ii) Related, imposing the standard CBCs leads to a different (unrenormalized) Brown-York stress-tensor (cf. Eqs. (103) and (106) of [hep-th/2409.07643]). From the perspective of CBCs, would it not be more natural to consider the Brown-York stress-tensors associated with CBCs?

i) We of course agree with the referee and our equations in fact show this. Please see (5.17). Please note that as we have pointed out (in few places) GHY b'dry term is just used as a reference point and W terms are added to move us away from this reference point. We added footnote 2 on page 12 to reemphasize again this point.

ii) That is an excellent point, and indeed, we have addressed it thoroughly in our analysis. We not only provide the explicit expression corresponding to the conformal boundary condition but also present analogous results for all the boundary conditions introduced throughout the manuscript. In fact, we go further by constructing the *renormalized* BY-EMT associated with each of these boundary conditions.

More precisely, in Section 6, we define a one-parameter family of hydrodynamic deformations that includes the conformal boundary condition as a special case. This family is characterized by equations (6.2) and (6.6), where $\tilde{\mathcal{T}}^{ab}$ denotes the rBY-EMT corresponding to boundary conditions induced by a function $G(\mathcal{T})$. The conformal boundary condition arises from the specific choice $G(\mathcal{T}) = \frac{2}{d}\mathcal{T}$, which we discuss in detail in that section.

Request 1: Highlight the new results of the manuscript.

Thank you for mentioning this. We have added a few paragraphs at the end of the introduction section, to highlight the new results of the manuscript.

Request 2: Update references, e.g., cite [hep-th/2412.16305] along with Refs. [87] and [88]).

Done.

Request 3: Respond to points (i) and (ii) of question (3) of the report.

Done.

References

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