Revision: SciPost Physics/2504.03411v1

**Report #2 by Anonymous (Referee 2) on 2025-5-28 (Invited Report)**

**The referee writes:**

**This manuscript considers the properties of one or a few bosonic impurities in a 1D weakly interacting Bose gas. The main novelty is that the impurity has two internal spin states, which are Rabi coupled: one internal state is interacting with the surrounding bosons and one is not. By carefully comparing with ab initio numerical calculations, the authors show that two simple analytical approaches based on a two-level approximation and an effective Hamiltonian both provide an accurate description of the system.**

**The topic of the manuscript is timely and interesting since there is presently a significant research activity exploring the physics of mobile impurities in a quantum degenerate environment. Before the paper is published, I however request the authors to consider the points below.**

**Our response:**

We thank the referee for their appraisal of our work but also their interesting comments and suggestions. Below, we provide a point-by-point reply to all issues raised as well as a list of changes at the end of the reply letter.

**The referee writes:**

**1. Is it really correct to speak about polarons in 1D? The existence of quasiparticles in 1D is not obvious. For instance, in arXiv:2504.17558 it was shown that a mobile impurity in a Fermi gas has zero quasiparticle residue. I presume the same holds for an impurity in a Bose gas? I suggest the authors discuss this in the manuscript.**

**Our response:**

This is a very interesting question. Indeed, in one-dimension the answer depends on the statistics of the host medium. Namely, it turns out that Fermi polarons do not exist in the thermodynamic limit while Bose polarons survive in the case of confinement as in our case. Specifically, in the case of an *interacting Bose gas*, the impurity dresses only $N_{\text{dress}} \sim n_0\xi$ bosons, where $\xi = \hbar/\sqrt{m_B g_{BB} n_0}$ is the healing length. Hence, the many-body overlap remains finite even in the thermodynamic limit. This can also be proven analytically within the Lee–Low–Pines + Gross-Pitaevskii equation (LLP + GPE) approach as it was demonstrated in [A. G. Volosniev and H. W. Hammer, Phys.

Rev. A **96**, 031601(R) (2017)], which can be rewritten as

$$Z = \exp\left\{\frac{\hbar c}{g_{BB}}\left[2\tanh^{-1}\left(\sqrt{1+\left(\frac{g_{BI}}{2\hbar c}\right)^2}-\frac{g_{BI}}{2\hbar c}\right)\right.\right.$$
$$+\ln\left(\frac{1}{2}\frac{g_{BI}}{2\hbar c}\left(\sqrt{1+\left(\frac{g_{BI}}{2\hbar c}\right)^2}-\frac{g_{BI}}{2\hbar c}\right)\right) \tag{1}$$
$$\left.\left.+1-\sqrt{1+\left(\frac{g_{BI}}{2\hbar c}\right)^2}+\frac{g_{BI}}{2\hbar c}\right]\right\},$$

where $c = \hbar/(\sqrt{2}m_B\xi)$ is the speed of sound. Therefore, $Z$ stays finite for any $g_{BI}$ provided $g_{BB} > 0$. In the weak-coupling limit $1 - Z \propto g_{BI}^2$; in the opposite limit $g_{BI} \to \infty$ the residue saturates at $Z \to \exp\big[-(2\ln 2 - 1)\hbar c/g_{BB}\big] > 0$, so the quasiparticle remains well-defined for all interaction strengths, within this approach.

Therefore, in our case where a harmonically trapped gas is used and finite-size effects cut off the infrared divergence reported in [F. Grusdt, *et al* New J. Phys. **19**, 103035 (2017)] the notion of the polaron is well-defined. A corresponding paragraph has been added to the manuscript (Sec. III B) summarizing the above arguments.

**The referee writes:**

**2. In Fig. 1, the case of zero Rabi coupling is labeled as a solid black line. However, I think the line is dashed.**

**Our response:**

We thank the referee for bringing this typographic error to our attention. It has been resolved in the revised version of the manuscript (see list of changes).

**The referee writes:**

**3. I don't understand the formula for the residue $Z$ given in the fourth line of page 8. Doesn't this formula give zero?**

**Our response:**

We thank the referee for bringing up this subtle point. In the revised version of the manuscript, the typographic error in the residue formula has been corrected and, for $\Omega_{\mathrm{R}0} = 0$, it is given by $Z = |\langle\Psi_{B+0\uparrow}|\hat{a}_{0\uparrow}|\Psi_{B+1\uparrow}\rangle|$. Here, $|\Psi_{B+0\uparrow}\rangle$ is the bath state in the absence of an impurity and $|\Psi_{B+1\uparrow}\rangle$ is the combined ground state of the bath and an interacting impurity.

**The referee writes:**

**4. The authors in general find very small deviations between the exact numerical calculations and the approximate theories. It would make the paper more interesting and suitable for SciPost, if the authors explore if this holds also for stronger impurity-boson interactions - especially on the attractive side where there is no phase separation. In particular, the authors state on page 14 that the effective Hamiltonian is accurate for $g_{BI} < g_{BB}$. Is this obvious for large and negative $g_{IB}$?**

**Our response:**

We agree with the referee that the prospective polaron properties are very interesting in the strongly attractive regime. However, one should emphasize that such ab-initio simulations are highly demanding since they require a substantially increased variational manifold to ensure numerical convergence especially for strong attractive interactions. Additionally, it is anticipated that an extended effective potential model would be necessary for the characterization of the emergent polaron state. This is indeed a highly intriguing research direction that we aim to undertake in forthcoming endeavors but certainly lies beyond the scope of the present work. Comments along these lines have been inserted in the revised manuscript to clarify the regime of validity of the present effective potential and highlight the importance to study polaron properties at strong attractive interactions in the future (see also the list of changes).

**The referee writes:**

**5. On page 18, the authors state that detunings $\Delta < -3\omega_B$ are suitable for studying the repulsive polaron. This probably depends on the value of $g_{IB}$. Should the condition involve the polaron energy instead?**

**Our response:**

Indeed the relevant detuning region depends on $g_{IB}$. Our statement in the main text refers to the $g_{IB} = 3g_{BB}$ case. In order to alleviate any possible confusion, in the revised manuscript, we explicitly state the discussed interaction region and further explain how the value of $\Delta$ can be estimated through the effective Hamiltonian approach.

**The referee's recommendation:**

**Publish (meets expectations and criteria for this Journal). Our response:**

We again thank the referee for their valuable comments and their recommendation.