Revision: SciPost Physics/2504.03411v1

Report #1 by Arturo Camacho Guardian (Referee 1) on 2025-5-18 (Invited Report)

The referee writes:

The manuscript "Competition of light- and phonon-dressing in microwave-dressed Bose polarons" by G. Koutentakis and co-authors studies the quasiparticle properties of a spinor impurity coupled to a scalar BEC confined in a one-dimensional harmonic trap. The internal states of the impurity are coupled by a light-field, one of the states is interacting whereas the second one is non-interacting. The authors adapt their numerical approach ML-MCTDHX (an ab-initio numerical approach) to study this system, this technique has been developed quite extensively in the group of some of the authors.

The topic of the manuscript is timely, the methodology robust, the results interesting and overall the manuscript is well-written. Before giving my final recommendations please find below my requested changes and questions.

Our response:

We thank the referee for their appraisal of our work and suggestions which helped us to provide further clarifications and improve our presentation. In the following, we provide a detailed response to all comments and append a list of changes.

The referee writes:

Requested changes

1. Some figures are quite crowded and difficult to interpret. For instance, in Fig. 1(a) and (b), it is hard to distinguish the differences between the plots, understand the role of N=2, and determine whether impurity-impurity interactions are present. The arrows in Fig. 1(b) are not particularly useful and add to the confusion.

Our response:

In the revised version of the manuscript we have splitted previous Figure 1 into two separate ones (now Figures 1, 2). Additionally, we provide a magnified view of the resonant region around $\Delta = -E_{1\uparrow}$ for $N = 2$ as panel (c) in Figure 1 such that the interaction shift (which is negligible in the discussed scales) becomes more visible.

The referee writes:

2. Perhaps I missed something, but one of the main conclusions seems to be that the residue increases with light-dressing. However, I couldn't find a figure that clearly demonstrates this. In fact, most of the results show a residue close to 1, so I struggle to understand how the impurity is dressed at all. (see next point)

Our response:

We thank the referee for this comment. Notice that one of our main findings is that the residue is captured by the experimentally accessible quantity $|\langle\hat{\boldsymbol{S}}\rangle|$. The behaviour of the latter within the different approaches we employ reveals information about the dressing of the polaron and the overlap of different involved states. This quantity when expressed in terms of the residue $Z(\Omega_{\mathrm{R}0})$ and Z_{eff} always results in a value of $0 < Z < 1$, indicating a dressed well-defined polaron. The limiting case $Z = 0$ indicates the absence of a quasiparticle since the many-body state is orthogonal to the free impurity state and $Z = 1$ marks the absence of a quasiparticle state because in this case the impurity state is independent of the bath. Neither of these scenaria are realized in our results, except for $\Delta \gg -E_{1\uparrow}$, where the non-interacting spin-$\downarrow$ state dominates, and thus $Z = 1$. To explicitly demonstrate the above and provide a direct connection to the corresponding many-body overlaps, in the revised manuscript we utilize the residua of the spin-projected many-body wavefunction. We have made several changes in the text to clarify these points and added a new Appendix C containing Fig. 9, where the residua of the spin-projected many-body wavefunctions are discussed (see also the list of changes).

The referee writes:

3. I also found the notation somewhat confusing, particularly in Section 3. My understanding is that the system can be described by a simple two-level model consisting of the non-interacting impurity state and the polaron state (in the limit of vanishing light-matter coupling). These two states then couple to the light field and hybridize. If this is correct, why is the residue not simply given by Eq. (7)?

Our response:

We thank the referee for this interesting question. While the two-level model is adequate for understanding the basic behavior of the system in terms of energies and populations of the spin-states, it is not able to correctly predict the behavior of more complex observables pertaining to the many-body state of the system. This can be readily seen by the fact that within the two-level system we expect $Z_{\uparrow} = Z$ (interacting spin state) and $Z_{\downarrow} = 1$ (non-interacting spin state) since it involves only the states $|\Psi_{0,+1/2}\rangle = |\Psi_{B+1\uparrow}\rangle$ and $|\Psi_{0,-1/2}\rangle = \hat{a}^{\dagger}_{0\downarrow}|\Psi_{B+0\uparrow}\rangle$. However, this is not the case within the many-body treatment as demonstrated by the newly added Figure 9 in Appendix C. Therefore, since the bath-impurity states within the spin-$\uparrow$ and spin-$\downarrow$ states change with respect to different system parameters (such as the detuning), the residue cannot be captured by the population of the spin-states given by Eq. (7). Comments along these lines have been added in the main text along with a new Appendix C in the revised manuscript (see also the list of changes).

The referee writes:

4. My main concern relates to the novelty and significance of the results, which connects to the previous comment. It appears that the light-matter coupling can be described by a simple two-level system, and that, in the explored regime, no particularly intriguing phenomena emerge. From what I understand, the main observable effect comes from Fig. 1(d), where there is a "drop" in S from 0.5 to 0.492—this seems rather small. Could the authors comment on the physical relevance of such a small difference? Is it experimentally observable? Also, did the use of the ML-MCTDHX method require any non-trivial extensions for this study?

Our response:

In view of the answers provided to questions 2 and 3 raised by the referee, we hope that they will be convinced about the interesting physics arising beyond the regime of validity of the two-level approximation, which we elucidate within the effective Hamiltonian model. Another focal point of our analysis is, of course, the competition of light and phonon polaron dressing as captured by the effective Hamiltonian model. On the other hand, achieving sensitivity of the order of $|\delta S| \lessapprox 10^{-3}$ or smaller is experimentally viable todate within Ramsey spectroscopy, see for instance the recent work by J. Etrych, *et al*, Phys. Rev. X **15**, 021070 (2025). Certainly, we anticipate increasing impact on the dressing toward the stronger attractive interaction regime which we aim to undertake in a future investigation.

Turning to the ML-MCTDHX approach, the one used herein corresponds to an extended implementation of an effectively scalar bosonic system coupled to an interacting spinor impurity. This required a few new technical implementations and optimizations regarding observables, especially regarding the fidelity calculations. The specifics of the structure of the wavefunction of the used setting are discussed in some detail within Appendix A and appropriate references are also provided.

The referee writes:

5. On page 13, the authors refer to the $1/k^4$ scaling related to Tan's contact to support

their claims. I don't fully understand this argument. As far as I know (though I may be mistaken), this scaling is a high-energy feature usually captured only with non-perturbative methods. While I agree that ML-MCTDHX is an ab initio approach, it is unclear to me how it could include the relevant two-body physics (e.g., Feshbach resonance physics) necessary to reproduce the $1/k^4$ scaling.

Our response:

We thank the referee for their comment. Here, we need to highlight that Tan's contact is only dependent on the scattering length and the two-body correlations emerging due to it. Regarding polaron physics, it already appears within perturbation theory see Table 2 in [F. Scazza, *et al*, Atoms **10**, 55 (2022)]. ML-MCTDHX has indeed access to this quantity since it takes all two-body correlations into account in a non-perturbative manner.

To avoid any possible confusion we amended the corresponding passage in the text. We now highlight the presence of additional short range contributions due to the bath-impurity interaction that are absent within the effective Hamiltonian treatment (see also the list of changes).

The referee writes:

Minor comments:

A. On page 2, second paragraph: it reads quasiparticletheories — a space is missing.

Our response:

We thank the referee for spotting this typographical error. It has been eliminated in the revised version of our manuscript.

The referee writes:

B. On page 2, the authors cite Refs. [69–82] as related to their previous work. Since this manuscript builds on those studies, could the authors clarify the relevance of each reference for the present work? They begin to do this on page 3, but the discussion could be made more explicit.

Our response:

We agree with the comment of the referee. To circumvent this issue, in the revised manuscript we have added a relevant discussion to highlight differences between the effective potential approaches used in the aforementioned referenced works.

The referee's recommendation:

Publish (easily meets expectations and criteria for this Journal; among top 50%) Our response:

We once more thank the referee for their suggestions that helped us to improve our presentation, and also for recommending our work for publication.