Dear Editor, Dear Reviewer

Many thanks for your appreciation, for reviewing our manuscript and for suggesting various improvements.

In *italic blue* you find the reviewer remarks, in red what we have actually implemented in the manuscript, and default style is used for our replies only to the reviewer. For clarity we have also numbered all the issues raised by the reviewer.

Reviewer 1

1. *The components of the rather crude position measuring apparatus are somewhat awkwardly named (e.g. Trans 2, Tile 1). Renaming them with some functional or position nomenclature might make the discussion of results more readable.*

Yes, we agree with the reviewer's comment and have updated the labeling accordingly (see Fig. 4). We now use "T1" and "T2" for the detectors measuring transverse compression, and "L1", "L2", and "L3" for those measuring longitudinal compression. Unlike the previous version, we no longer assign separate numbers to the top and bottom rows of detectors. Instead, we now consistently refer to them as, for example, "T1 top", "T1 bottom", or "T1 coincidence".

2. *Continuation of the tile timing simulation in fig. 5 beyond 6 μs would be interesting to see; alternatively, provide an explanation for the cut-off.*

In the original plot, the black line (tile timing simulation) of Fig. 5 (bottom) also extended to 10 μs, but it was not visible at late times because it perfectly overlapped with the red curve. There was no cut-off. Both curves had been normalized to the same value at late times. To make them distinguishable, and to be consistent with the red curves shown in the top and middle panels and with the description in the text (see beginning of page 11), we have now normalized the red curve to match the average of the data at late times (which in this case is consistent with the maximum).
It is important to note that the red curves in Fig. 5 are not fits to the data—given the significant discrepancies—but are simply normalized to match the maximum of the measured data and are shown only to illustrate the timing mismatch.

3. *It would be useful to further explain the reference to muons "flying by". Is something missing from the simulation causing the disagreement at later times; even with the introduction of extra parameters the agreement of the simulations with data isn't great.*

The simulation, as far as we can tell, are complete. To clarify the doubts of the reviewer associated to the "flying by" we significantly improved the whole paragraph: The previous paragraph:

"*The decrease in counts at later times in the T1 and T2 detectors' time spectra is due to muons "flying by" regions with maximal detection efficiency (see Fig. 4). In contrast, the counts in the L2 spectra remain steady at later times, indicating that the muons have* *reached and come to rest at the target tip, where detection efficiency is maximal. Without transverse (y-direction) compression, the majority of the muons would impactthe top or bottom target walls before reaching the target tip, which would limit the counts in he L2 detectors. Additionally, it would reduce the "fly-by" effect (decrease at late times) in the T1 and T2 detectors. The large number of counts in the L2 spectrum indicates not only that muons are drifting in x-direction but also that rather efficient transverse compression is indeed taking place.*"

has been replaced by:

"The decrease in counts for t > 2.5 µs in the T1 time spectrum is due to muons leaving the region of maximal detection efficiency for the T1 coincidence, which is located around x ≈ –5 mm for y ≈ 0 (see Fig. 4). A similar decrease is observed in the T2 time spectrum for t > 4 µs, indicating that muons pass through and eventually leave the region of maximal detection efficiency for the T2 coincidence, located around x ≈ 10 mm. In contrast, the lifetime-compensated counts in the L2 spectrum remain constant at late times. Indeed, as shown in Fig. 4 (Right), muons drifting in the +x-direction are unable to exit the region of highest detection efficiency (for the L2 coincidence) located at x ≈ 22 mm, as the target ends up at x ≈ 24 mm. At late times, all muons must eventually come to rest on one of the target walls, such that the lifetime-compensated time spectrum becomes flat. Without transverse (i.e., y-direction) compression, most muons would hit the top or bottom target walls before reaching the tip of the target, resulting in much less L2 coincidence counts and a less pronounced decrease in the T1 and T2 signals. The large number of counts observed in the L2 spectrum therefore demonstrates not only that muons are drifting in the x-direction, but also that an efficient transverse compression is indeed taking place."

4. *It appears that fig. 6 contains the same data for Tiles 2,5 as fig. 5; were simulation done for the other tiles indicating some level of agreement? In Fig. 9 the B=5 T data/ simulations disagree by a lot; is there an explanation?*

The reviewer is correct that Fig. 5 and Fig. 6 partially display the same underlying data. However, the figures serve different purposes: Fig. 5 highlights the transverse compression and the subsequent drift in the x-direction, while Fig. 6 focuses on the compression along the x-direction itself. In both cases, the former Tile 2,5 detectors (now labeled as the L2 coincidence) are used to illustrate these respective processes.

The simulations for the other tiles (currently T1 and T3) are in good agreement with the data. The reviewer is correct in noting that we did not emphasize this point sufficiently. To address this, we have now included the corresponding fits in Fig. 6. And we added "as expected from simulations" at the end of the paragraph".

Regarding the remaining discrepancy between simulation and experiment, we unfortunately do not have a definitive explanation. As described in the manuscript, we have varied all relevant physical parameters within realistic bounds, placing greater weight on measurements with higher statistical precision. However, despite these efforts, the agreement could not be further improved.

5. *The conclusions present an estimate of the overall compression efficiency; it would be useful to show the factors and uncertainties that went into those estimates.*

We agree with the reviewer. Hence we have expand significantly the previous paragraph which was reading:

"*This efficiency allows an estimate of the overall compression efficiency of up to 10^{-4}, taking into account the coupling efficiency into the solenoid, the stopping probability in the target, muon decay and losses during extraction and re-acceleration*"

to

"This efficiency allows us to estimate the overall performance of the muCool compression scheme. The total efficiency is estimated by considering each sequential stage — starting from muon injection into the solenoid and target, followed by compression, extraction, and final re-acceleration — with the following approximate efficiencies for each step as obtained from GEANT4 simulations: injection into the solenoid (50%); transmission through the target entrance window (50%); muon stopping probability in the active region of the helium gas target (0.6%); compression efficiency, excluding muon decay (90%); survival during the~ 5 μs compression time, accounting for muon decay (8%); extraction efficiency, excluding muon decay (80%); survival during the ~ 1.5 μs time needed for extraction and propagation to the re-acceleration stage (50%); coupling into the re-acceleration stage (80%); and re-acceleration to 10 keV followed by extraction from the magnetic field (50%). Multiplying these factors yields an overall efficiency of approximately 2 × 10–5. This estimate assumes operation with the HIMB beam [20, 21] at a momentum of 28 MeV/c and the implementation of a single muCool target. Higher overall efficiencies —up to 10–4—are expected when using alternative beamlines such as πE5 or πE1, or by employing an array of muCool targets."

Best regards

muCool collaboration