Reply to the Referee's report

Dear Editor and Referee,

First of all, we would like to thank you for the attention to our paper and valuable comments and suggestions.

Below we put our answers to the points raised by the Referee.

Point 1 is indeed a misprint which we will correct.

Point 2 — we will replace a vague sentence "and multiple other references." by "(for a recent review see, for instance [2505.23895])" and will insert a corresonding reference.

Point 3 — we agree that adding numbers to equations will simplify orienting in the manuscript. All the omitted numbers will be added. Also missing commas and full stops will be added after equations for clarity.

Point 4 — Notations after equation (16) (which WILL BECOME equation (20) after renumbering) G_t means $\partial_t G$ and this will be clarified.

Point 5 is indeed a misprint which will be corrected.

Point 6 — Explanation between equations (18) and (19) (which WILL BE equations (22) and (23) after renumbering)

We will add the following more explicit phrase after mentioning the asymptotic behavior: This functional dependence introduces two branch points in the complex ω-plane at $\omega = \pm\mu$ (or $\pm a$ in our notation), which generate the branch cut singularities of the Green's function.

Point 7 — Figures will be improved by enlarging corresponding fonts.

Point 8 — Extra clarifications regarding the convergence.

By a convergence test at some chosen data points, we use continued fraction with depth 100 so that the relative error is less than 10^{-5}.

We will introduce this extra note in the text as well add more details of the method used to make a reader aware of them.

Sincerely,
the authors