Response to Referee Report: Scipost

Dear Editor and Referee,

We would like to thank the referee for a careful reading of the manuscript and the comments. We have taken into account the recommendations made by the referee in their report, and we would like to respond in the following paragraphs.

- **Point #1:**

  In the submitted version of the manuscript to scipost journal, we had dedicated subsection 3.5 to discuss about the holographic duals of the three probe M5 examples that we discussed in section 3. In this subsection, we focused specifically on the first example $\Phi_1 = 0$; $Z_2 = 0$; to recover the defect in the boundary gauge theory by taking a large value limit of the AdS_7 radial coordinate. The steps discussed here also apply to the other two examples to recover the respective defects of the same features and symmetry.

  Changes made in the revised version:

  We have edited subsection 3.5. We have sharpened the comments made in the discussion in 3.5. We have described the features of the dual defects a bit more and their connection to the *rigid* surface defects of Gukov-Witten in reference [4]. Although the discussion in this section is not a proof of the duality with the *rigid* defects, we have highlighted the features that make the basis of our expectations.

- **Point #2:**

  In this manuscript, we have found a new class of M5 brane solutions that preserve at least 1 supersymmetry from the 11d background spacetime geometry of $AdS_7 \times S^4$. There have been many papers in the past that have discussed half-BPS solutions solving the κ-symmetry constraint equation. But there have been very few papers that have done it for the M5 branes that preserve so little amount of susy. In this article, we work in a special vielbein frame that allows us to express the AdS_7 part as a $U(1)$ Hopf fibration over a Kaehler manifold $\widetilde{\mathbb{CP}}^3$, and the S^3 inside the S^4 part as a $U(1)$ Hopf fibration over a Kaehler manifold $\mathbb{CP}^1$(we had put this detail in appendix A). Due to this choice of frame veilbein, in section 2 of this manuscript, we have been able to derive our general solution for probe M5 in terms of two arbitrary holomorphic conditions in equation (2.25). The analysis that we do in Section 3 is for the simpler case examples coming from the general condition in equation (2.25). In section 3, we have done the κ-symmetry calculation in detail for the half-BPS example $\Phi_1 = 0; Z_2 = 0;$ to emphasise two new features that we noticed in this work:

i) The significance of the position of the probe M5 on the S^4 polar coordinate θ direction. The detailed calculation here shows how the probe M5 is not allowed to carry the new fluxes from section 2(obtained in eqn (2.19)) at $\theta = \pi/2$ location.

ii) To highlight how the supersymmetry breaks differently when different components of the 3-form flux field h are turned on. For the flux components chosen in subsection 3.2, susy breaks by an additional factor of 1/4. And for the flux components chosen in subsubsection 3.2.2, susy breaks by an additional factor of 1/8.

Changes made in the revised version:

We have reduced the number of steps between equations (3.6) and (3.9). We have only kept two steps before writing the projection condition for the example $\Phi_1 = 0; Z_2 = 0;$. The first of those steps is to define the exponential factor M that we use repeatedly in this section. And the second step highlights the location of the brane at $\theta = \pi/2$. We also restructured many equations in this section, which were written ill-formatted manner previously. We have also relegated the detailed calculation of the example $\Phi_2 = 0; Z_2 = 0;$, in subsubsection 3.3.1, to a new appendix D.

- **Point #3:**

  Changes made in the revised version:

  We have added a new table(#2) where we summarise the results of the three half-BPS examples discussed in section 3. In the last two sets of rows of this table, we have also written the summarised result of the general 1/32 BPS solution obtained in section 2, so that a direct comparison can be made between all three examples that descend from the general result of section 2. We have also edited the heading title of subsection 3.2, subsubsections 3.3.1 and 3.3.2, so that their relation with the two conditions in eqn 2.25 becomes more apparent.

We hope that after the above revisions, the manuscript is acceptable for publication.