Response to Referee Report 2

We would like to thank the referee for their careful reading of the manuscript and their useful comments and suggestions, and for pointing out directions for further development. Below, we provide point-by-point responses to the referee's comments.

Referee Comment 1

"This is a very interesting mathematical problem in its own right. However, it will be good if the authors can add few sentences to motivate this from a physical point of view. I have a feeling that this is somewhat lacking in the paper."

Response: We thank the referee for making this point. We have added the following lines at the end of the second and third paragraphs of section 1.3 (in the introduction) to highlight some of the physical implications of our results.

Line added at the end of the second para in section 1.3: *As correlation functions capture the response of a theory to sources, our results are needed (together, of course, with knowledge of the correlators as analytic function of z and $\bar{z}$), to determine the physical response of a CFT on $S^1\times$ time to arbitrary sources as a function of angle and time.*

Line added at the end of the third para in section 1.3: *It is physically interesting that several causally distinct configurations sometimes land on the same sheet of the correlator. This tells us that different (and symmetry unrelated) physical experiments sometimes have identical answers. We leave the interesting problem of understanding this observation from a physical viewpoint to future work.*

We hope this addition goes some way to ameliorating the referee's concern on this point.

Referee Comment 2

" In section-2 that authors show that "path-independence" leads to the condition that the spin of all the inserted operators should be integers. This condition can also be obtained from the analysis of the Euclidean CFT correlation functions. Do the authors expect that a similar analysis of higher point Lorentzian correlators will give rise to other constraints on the CFT data which may not be (easily) accessible from Euclidean CFT?"

Response: We thank the referee for this important question. We had initially hoped it would turn out that the physical requirement - that different paths leading to the same final insertion location must yield the same final correlator - would lead to nontrivial new constraints on four-point

functions. However, we regard our proof that four-point functions are automatically path independent (as long as all insertions have integer spins) as negating our initial hope; path independence imposes no nontrivial constraints on four-point functions. While it is possible that one might encounter a surprise while investigating higher point functions, we no longer expect this to be the case.

In response to this question, we have added the following new statement at the end of subsection 1.2: "It is also a disappointment because it tells us that the requirement of 'path independence' is automatic, and imposes no new general constraints on four-point functions of operators with integer spins."

Once again, we thank the referee for their thoughtful feedback which has improved our paper. We hope the paper is now suitable for publication.

Sincerely,
Suman Kundu, Shiraz Minwalla, Abhishek Navhal