Response to points raised by Referee

1.*Referee:* *Check for FPT – The authors show gradual buildup of the correlations order by order in perturbation theory. This might lead the authors to think that this build up is a property of the Magnus expansion. The authors, to counter this point, could, for example use Floquet Perturbation theory (or Magnus in a rotated frame) which has a wider validity range. I think this point needs to be checked to ascertain the universality of the results.*

Authors: We thank the referee for this insightful suggestion, which prompted us to further substantiate the universality of our central finding – the path superposition principle. We agree that relying solely on the convergence of the Magnus expansion (ME) could raise concerns about its method-dependence. Our primary evidence for universality stems from the **consistent quantitative agreement** between our second-order ME results and **exact numerical simulations** across various systems, including different lattice geometries and quench protocols. This robust agreement strongly suggests that the superposition law captures the underlying physics of correlation buildup during the quench dynamics, rather than being an artifact of the perturbative method. Regarding the suggestion to employ Floquet Perturbation Theory (FPT), we note that our work focuses on **single-quench dynamics**, for which the ME is a natural and suitable analytical tool. FPT, while powerful for periodically driven systems, is formally designed for Floquet steady states and may not be directly applicable or necessary for our transient quench scenario. The excellent performance of ME in our context justifies its use. Nevertheless, exploring whether the superposition law extends to periodically driven systems using FPT is a compelling future direction. We have therefore added a discussion in the conclusion: "*Investigating the persistence of the path superposition principle in periodically driven systems, for instance using Floquet Perturbation Theory [New J. Phys. 20, 093022 (2018)], represents an important future research direction.*"

2.*Referee:* *Comment on domain of validity of the Magnus expansion – The use of Magnus expansion means that the results can only be shown for high frequency (even when we discuss finite time). The authors need to discuss this more clearly. In particular, they need to point out the radius of convergence of this expansion in the regime they are interested in.*

Authors: We thank the referee for raising this crucial point regarding the domain of validity of the Magnus expansion (ME). Our application of the ME indeed differs from its conventional use, and we appreciate the opportunity to clarify this distinction. The referee is correct that when the ME is used to derive a static **Floquet Hamiltonian** for a periodically driven system, its convergence requires a high-frequency condition ($\Omega \gg$ energy scales) to accurately capture the long-time stroboscopic dynamics. However, our work employs the ME in a fundamentally different context. We study a **single, non-periodic quench** rather than a periodic drive. Here, the ME is not used to find a Floquet Hamiltonian but to construct a **finite-time evolution operator** $\hat{U}(T)$ directly. The convergence criterion for this application is a **short-time condition**, not a high-frequency one. The standard sufficient condition for the convergence of the ME series is $\int_0^T \|H(t)\|_2 \, dt < \pi$, which, for our quench protocol, implies an upper bound on the product of the quench amplitude and duration, $|\Delta\delta| \cdot T$. Therefore, the "radius of convergence" in our context is defined by the **finite quench time T being sufficiently short**. We provide two key evidences that our chosen duration ($T = 0.5\mu s$) lies well within this convergent regime: (i) **Empirical Numerical Agreement:** As shown throughout the manuscript, our second-order ME results are in quantitative agreement with exact numerical simulations across various lattice geometries. (ii) **Theoretical Self-Consistency:** As rigorously shown in Appendix B, our expansion correctly yields null correlations for the non-interacting case ($U = 0$) at any truncation order, satisfying a fundamental physical constraint and validating the internal consistency of our formalism. For $U \neq 0$, we have systematically verified the validity of our approach. We find that quantitative agreement with exact numerics (see Fig. 8 in Appendix A) is achieved only when our expansion incorporates all key physical processes: (i) the shortest path; (ii) couplings from neighboring sites; (iii) temporal interference from interaction time-ordering. In summary, the high-frequency limit is not pertinent to our single-quench problem. The ME, governed by a short-time convergence criterion, is exceptionally well-suited for providing an accurate and analytically transparent description of our quench dynamics, successfully revealing the universal path superposition law.