Dear Editor,

We thank the referees for their thorough revision and relevant comments. We hope they find the revised version of the manuscript satisfactory. We have carefully considered their comments and suggestions, and have made corresponding revisions throughout. We believe that both the manuscript and this response now convey the motivation behind our work more clearly, and we hope it will be recognized as a valuable contribution to SciPost Physics. Thank you for your consideration.

Please find attached our point-by-point response to referees' comments and questions. A list of the changes made in the revised version of the manuscript is provided below.

Ljubljana & Garching November 4, 2025
Michele Coppola, Mari Carmen Bañuls, Zala Lenarčič

Referee: 1

Strengths

1. *Provides a detailed and accurate data shedding new light on the efficient long-time simulation of local observables in quantum systems.*

2. *Raises new and interesting questions (for example - the remarkable accuracy of the Markovian approximation to the influence functional in certain cases)*

3. *Is clearly written and makes transparent connection to the preceding literature in the field.*

We thank the referee for the positive comments.

Weaknesses

NA

Report

The is a topical and substantial piece of work containing mainly numerical analysis of the accuracy of the Markovian approximation to the influence function in 1d systems. It addresses a set of outstanding questions in a clear and transparent way and raises some interesting questions for future analysis.
The results themselves are remarkable for the accuracy with which this Markovian approximation captures the dynamics of local observables. Perhaps this was not surprising to the authors - it is to me.
I have no criticism of the work per se, but it does raise a number of questions for me that the authors may consider discussing. That the work raises such questions is a strength rather than a weakness.

We thank again the referee for the positive feedback.

Requested changes

The authors might consider adding a brief discussion of the following points:

1. *Once the influence function settles, a constant approximation to it for later times does a good job of approximating local observables. I would have liked a little more discussion about the nature of this steady-state influence function: does it characterize the influence of a thermal system at the energy density considered?*

   The referee raises an interesting point. Intuitively, indeed, one would expect that, in the generic case, the thermalizing dynamics of the subsystem after a long time will be similar to the one induced by a thermal bath, at a temperature determined by the energy density of the chain. For the integrable case, conserved charges may impose further constraints to the effective description of the bath at long times. In order to probe this point in more detail, one could test whether the reconstructed Lindblad operators for the chaotic case, at times close to the steady state, obey the detailed balance condition. However, even for the chaotic case, we still see strong non-Markovian effects and not exactly stationary behaviour. So we might need to analyze the maps at even longer times, and/or with initial state of higher energy with supposedly more Markovian behavior. Another controlled study along these lines would be to initialize the left and right environment not in pure but thermal states. We thus postpone such analysis to a follow up work, where we will controllably study the effect of different choices of initial states for the environment.

2. *On a related point, the authors mention that they actually find multiple fixed points the choice between which does not affect local observables. A little more discussion of the nature of these different influence functions would be instructive - should they be thought of as different parametrizations of the influence of the same steady state or is something else going on?*

   The referee is correct; non-uniqueness of the numerical solution seems to be a feature of both, long and intermediate times. Notice that the non-uniqueness affects only our reconstruction of the channel, and not the left and right environments found by our TN calculation. So it seems that while there should be a unique influence function representing the environment, there are simply several time-local attempts to approximate it. It is hard to distinguish whether this as a consequence of singular nature of dynamical maps, the shallowness of our optimization landscape (possibly also related to the choice of distance) or a finite number of optimization steps. Nonetheless, the general features of different numerical optimizations are stable (e.g., appearance of negative rates) and observables well represented. Corresponding clarifications have been added to Sec. 5 in the manuscript to address the referee's request.

3. *The information lattice work of [12], suggests (if I understand correctly) that retaining information on a slightly longer scale than that of interest might be sufficient. In this context it might be useful to show the single site observables in the $l = 2$ case especially in the non-integrable case in the region where otherwise the simulation is slightly off.*

   In Fig. 1, we plot the difference between single-site observable σ^x dynamics obtained from reconstructed one-site or two-site time-local maps (R) and the corresponding one-site/two-site quasi-exact dynamical maps from light-cone contraction (TN). We can see that there is no consistent observation; for some models, the single site reconstructed map outperforms the two-site map, and vice versa. We agree with the referee that, according to SciPost Phys. 13, 080 (2022), two-site maps retain more information from nearest-neighbor correlations, but they do require more parameters to be optimized in our protocol. In order to observe the proposed effect, we might need to increase the number of optimization steps or further increase the support considered (which was the case in SciPost Phys. 13, 080 (2022).)

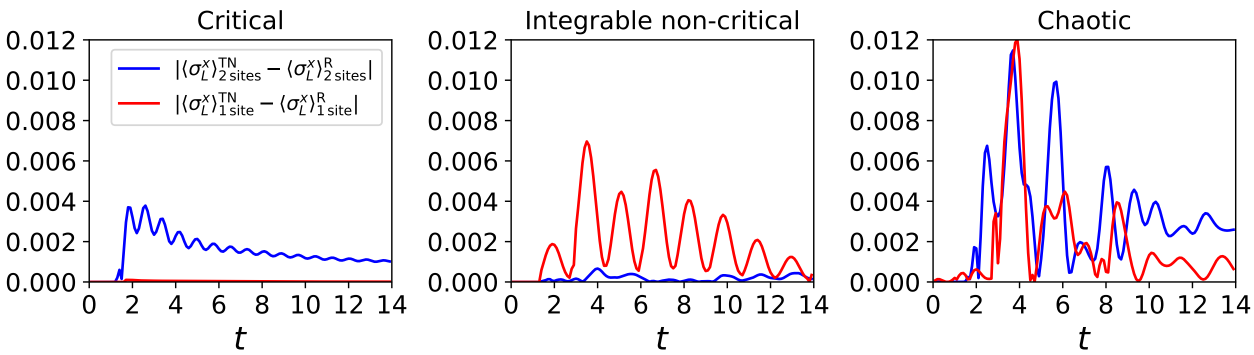

Figure 1: Difference in the modulus of single-site observables between one-site and two-site quasi-exact dynamical maps (TN) and the reconstructed (R) evolution.

Recommendation

Publish (surpasses expectations and criteria for this Journal; among top 10%*).*

We are grateful to the referee for the valuable comments. We have revised the manuscript in light of the referee's suggestions and we hope that the changes fully address the referee's concerns.

List of changes

- In Sec. 3 (page 6), we add "matrix product operator (MPO) ansatz"
- In Sec. 4 (page 8), we specify: "In addition, we would like to emphasize that the choice of the Frobenius norm in (10) is primarily motivated by practical reasons. Although alternative definitions of the loss function based on the trace norm and the distinguishability of quantum states could in principle be employed, they are typically of impractical use because of the high computational cost in minimization. For this reason, the Frobenius norm (10) was chosen for its efficiency."
- In Sec. 5 (page 9), we add: "Since we aim to obtain Liouvillians Λ_t that are as continuous in time as possible, we use the optimized $h_i(t-dt)$ and $c_{ij}(t-dt)$ as input values for the subsequent optimization at time t. However, we should note that a different initialization could result in another solution of a similar quality. We believe this is due to existence of several (local) minima of the loss function, possibly a consequence of singularity of the dynamical map and/or the norm used in the loss function."
- In Sec. 6 (page 14), we specify: "While there are different definitions of Markovianity in the literature, the RHP criterion states that a dynamical map is Markovian if and only if it is CP-divisible."
- In Sec. 6 (page 15), we specify: "On the other hand, for the non-critical integrable model, there are quasi-particles with zero or small velocities that transverse through the subsystem on longer timescales, and in this way contribute to the non-Markovian features of the dynamics. The chaotic model does not allow for a quasi-particle description thus the source of (non-)Markovianity is different there. Our observation with lower-level of non-Markovianity in the chaotic model compared to the gapped integrable case seems to be consistent with Ref. [60], which argues that, for generic dynamics and small enough subsystem, the evolution of the subsystem is almost Markovian with high probability."
- In the conclusions (page 18), we add: "An interesting research direction could be to investigate whether a more Markovian dynamics emerges from initial states at higher energies and, if so, to characterize the thermal nature of the environment by checking the detailed balance condition at the level of the learned Lindblad operators."