Referee reply for "Non-Stabilizerness of Sachdev-Ye-Kitaev Model"

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Reply to Referee 1 (***Pengfei Zhang***)

I believe these results are of particular interests to the field of quantum dynamics and SYK-like models: They provide valuable examples for the behavior of Majorana spectrum/stablizer Renyi entropy in concrete many-body systems. In addition, given the close relationship between SYK models and gravity, this may inspire the study of stablizer Renyi entropy in holography. Therefore, I'm happy to suggest the publication of this work in SciPost Physics.

Response: We sincerely thank the referee (Dr. Pengfei Zhang) for the positive assessment of our work and for highlighting its relevance to the study of quantum dynamics and SYK-like models. We are particularly pleased that the referee emphasized the potential implications of our results for understanding the behavior of the Majorana spectrum and stabilizer Rényi entropy in concrete many-body systems, as well as their possible connection to holographic studies. We appreciate and thank again the referee's recommendation for publication in SciPost Physics.

In the following, we address the referee's two minor comments in detail.

Comment 1: The Majorana spectrum is auctually the expectation of a Majorana string. For the SYK2 model, the expectation should satisfies the Wick's theorem, which relates a generic Majorana spectrum to two-point correlators. Does this directly lead to the observed exponential Laplace distribution directly?

Response: We thank the referee for this insightful comment and question. The referee is indeed correct that the Majorana spectrum corresponds to the expectation values of all Majorana strings. Since the SYK_2 model is non-interacting, Wick's theorem applies, and the expectation value of a Majorana string can be expressed as a determinant of the covariance matrix constructed from all possible two-point correlators. However, the Majorana spectrum involves sampling over an exponentially large number of such determinants, and hence this does not directly imply the observed Laplace distribution. We believe that the Laplace distribution is a generic feature of integrable random Gaussian systems, such as the eigenstates of the quadratic SYK_2 Hamiltonian.

Comment 2: In the main discussion, authors consider the complex SYK model. Could authors comment on this choice? In particular, is there any difference when considering the Majorana SYK model? Does charge conservation play any role?

Response: As the referee correctly pointed out, the complex SYK model possesses a $U(1)$ symmetry, leading to particle-number conservation. This allows us to diagonalize the Hamiltonian within specific particle-number sectors, such as the half-filling sector considered in our work. Since the SYK model contains disorder, disorder averaging is required, and using the complex SYK model provides a practical advantage over exact diagonalization. Moreover, due to particle number conservation and fermionic parity $D/2 = 4^N/2$ expectation values are zero. Therefore, the computational complexity of evaluating the stabilizer Rényi entropy (SRE) for the complex SYK model is relatively better compared to the Majorana SYK model numerically.

More importantly, in our work we also investigated the unitary dynamics of the SYK (and SYK_2) model. For studying unitary dynamics, it is natural to work with the complex SYK model, which possesses a $U(1)$ symmetry and allows us to choose stabilizer initial states such as a charge-density-wave state with a fixed particle number. This approach is equivalent to that of Ref. 73 (arXiv:2502.01431), where the authors studied the SYK model in the spin representation within the total $S^z = 0$ sector—corresponding precisely to the half-filling sector of the complex SYK model.

We do not expect significant differences when considering the Majorana SYK model. Comparing the ground-state SRE of our complex SYK model with that of the Majorana SYK model in Ref. 74 (arXiv:2502.03093), we find that the SRE values are of similar order of magnitude.