Referee reply for "Non-Stabilizerness of Sachdev-Ye-Kitaev Model"

Surajit Bera[1], M. Schirò[1]

[1] JEIP, UAR 3573 CNRS, Collège de France, PSL Research University, 11 Place Marcelin Berthelot, 75321 Paris Cedex 05, France

November 12, 2025

Reply to Referee 2

I think this is an interesting addition to the literature because these results potentially provide a way to characterize the impact of interaction on magic. Moreover, these results could also be of interest in the low dimensional quantum gravity community due to the duality of the interacting SYK model to JT gravity. Moreover, this community is currently interested in quantum information concepts and idea.
I am glad to accept the paper for publication after address the following comments.

Response: We sincerely thank the referee for positive assessment of our work and for emphasizing its relevance for characterizing the impact of interactions on magic. We are also pleased that the referee highlighted the potential interest of our results for the low-dimensional quantum gravity community, given the duality between the interacting SYK model and JT gravity, and the community's current focus on quantum information concepts. We appreciate and thank the referee's recommendation for publication after addressing the remaining comments, which we respond to below.

Comment 1: I am surprised that computationally the SYK_2 does not allow a short-cut to reach much larger sizes. Even numerically, the application of the Wick theorem should make possible to reach much larger sizes. Could the author comment on this?

Response: We agree with the referee's expectation that Wick's theorem holds for the non-interacting SYK_2 model and indeed makes the computation of local observables straightforward, allowing access to very large system sizes. However, the computation of the stabilizer Rényi entropy (SRE) remains inherently exponential, since it requires evaluating the expectation values of an exponentially large number of Majorana string operators. While in the non-interacting case (SYK_2) these expectation values can be expressed as determinants of the covariance matrix of all possible two-point correlators using Wick's theorem, one would still need to sample all of these exponentially many determinants, which is computationally very demanding. To compute the SRE efficiently in larger systems, one would need to employ perfect sampling techniques for Majorana strings, as described for Gaussian fermions in Ref. 41 by M. Collura *et al.* (arXiv:2412.05367). Moreover, the main focus of our work is to compute and compare the SRE and the Majorana spectrum of the many-body chaotic SYK model, with the non-chaotic SYK_2 model serving as a reference for a non-chaotic many-body system. For this purpose, we restrict our analysis to system sizes accessible through exact diagonalization, even for SYK_2.

In our revised manuscript (see page 10 in new version), we add the following text to incorporate the referee's comment and to make more clear message of our work:

" We note that for random gaussian fermions an efficient algorithm has been proposed to compute the SRE [41], which could be in principle adapted to compute the Majorana spectrum. Here, we limit our analysis to small system sizes to perform a fair comparison with the SYK4 model, which is the main focus of our work."

Comment 2: I would also ask the authors to provide a more detailed description of similar calculations in the recent literature. They mention overlap with 72,73. They should elaborate. More specifically, they should comment to what extent their results are universal at least in the context of Fermionic theories. Is expected a linear scaling with N of the SRE in all fermionic systems? Is a Gaussian and an exponential distribution generic for interacting and integrable fermions?

Response: We thank the referee for this question and address it only insofar as our work overlaps with the cited references.

Ref. 72. Ref. 72 studies the SRE and entanglement measures for a combined Majorana SYK and SYK_2 setting. Our overlap with Ref. 72 concerns the ground-state SRE: comparing our complex SYK results at system size $N = 14$ with their Majorana SYK results at $N = 28$ (two Majoranas $\equiv$ one complex fermion) suggests that the Majorana SYK model exhibits a slightly smaller SRE than the complex SYK model, though both are of comparable magnitude.

Ref. 73. Ref. 73 investigates both unitary and monitored dynamics of the SYK model in the spin representation within the total $S^z = 0$ sector, which corresponds precisely to the half-filling sector of the complex SYK model studied here. Their steady-state SRE under unitary dynamics from a Néel initial state is of the same order as our steady-state SRE obtained from a charge-density-wave initial state.

In our work, we examined both (i) the ground-state SRE (overlapping with Ref. 72) and (ii) the unitary dynamics (overlapping with Ref. 73). In addition, we analyzed the Majorana spectrum, which contains information beyond the moment-based SRE.

The linear scaling of SRE is expected for generic fermionic states (away from stabilizers), since SRE quantifies the entropy of the expansion in the Majorana/Pauli basis. Its magnitude is typically larger in fermionic systems than in spin systems due to non-local Jordan–Wigner strings placing fermionic states further from stabilizers. Consistent with this, we find that strong interactions in SYK enhance SRE relative to the quadratic SYK_2 case.

Regarding majorana spectrum, a Gaussian distribution of the Majorana (equivalently, Pauli) spectrum is generic in interacting chaotic systems by quantum typicality argument. By contrast, we find an exponential (Laplace) distribution for the integrable SYK_2 model, which we believe is a generic feature of random Gaussian states (to which SYK_2 eigenstates belong), and we do not expect it to arise in integrable models without randomness.

Finally, we thank the referee for raising this valuable point. We have incorporated the above clarifications into the revised manuscript (in main text and the *Note Added*).

In the main text, we add the following texts (see page 10 in new version) :

"Such a feature was found also in non-integrable spin chains [34], and is a generic feature of chaotic many-body system".

"Even with these small sizes, the agreement with the Laplace distribution for the Majorana spectrum of the SYK_2 *model is excellent. Our results suggests therefore that the Laplace*

distribution is a generic feature of the Majorana spectrum for random Gaussian states."

Comment 3: Why do the author study the SYK with Dirac fermions instead of the flavor with Majoranas? I would expect that the Majorana one is more interesting because larger sizes are available and because it is more directly related to Pauli spins.

Response: We thank the referee for this question. We study the SYK model with Dirac fermions instead of the Majorana flavor because the complex SYK model (Dirac fermions) possesses a $U(1)$ symmetry that leads to particle-number conservation. This allows us to diagonalize the Hamiltonian within specific particle-number sectors, such as the half-filling sector considered in our work. Since the SYK model contains disorder, disorder averaging is required, and using the complex SYK model provides a practical advantage over exact diagonalization. Moreover, particle-number conservation together with fermionic parity implies that $D/2 = 4^N/2$ expectation values of Majorana operators vanish *exactly*. Therefore, the computational complexity for the complex SYK model is relatively reduced compared to its Majorana counterpart.

More importantly, in our work we also investigated the unitary dynamics of the SYK (and SYK_2) model. For studying unitary dynamics, it is natural to work with the complex SYK model, which has a $U(1)$ symmetry and allows us to choose stabilizer initial states such as a charge-density-wave state with a fixed particle number. For time evolution, full diagonalization of the Hamiltonian is not required. Since the dynamics preserve particle number, we only need to diagonalize the Hamiltonian within a fixed particle-number sector (half-filling). Otherwise, performing time evolution without exploiting the $U(1)$ symmetry would be numerically much more demanding. As we noted in our response to Comment 2, this approach is equivalent to that of Ref. 73, where the authors studied the unitary dynamics of the SYK model in the spin representation within the total $S^z = 0$ sector—precisely corresponding to the half-filling sector of the complex SYK model.

Comment 4: The SYK references require some tweaking. When referring to SYK the first time, [45] is irrelevant and [49] should be cited and Kitaev talk should likely be first. SYK models for Dirac fermions in the context of RMT were studied much earlier than [43] but I do not want to be too intrusive with respect to citations. When referring to black holes, the authors should refer to seminal papers and not to what look like a review. The relevance to gravity/black holes is already in Kitaev talk and also in [49]. There were several early papers that pointed out this relation. For instance, https://arxiv.org/abs/1611.04650 https://arxiv.org/abs/1610.03816
https://link.springer.com/article/10.1007/JHEP08(2017)136
https://journals.aps.org/prx/abstract/10.1103/PhysRevX.5.041025
The authors could take a look a choose a few.

Response: We thank referee for pointing out these errors, and suggesting the correct references. We now have changed the references accordingly in our new version.