# Referee reply for "Non-Stabilizerness of Sachdev-Ye-Kitaev Model"

Surajit Bera[1], M. Schirò[1]

[1] JEIP, UAR 3573 CNRS, Collège de France, PSL Research University, 11 Place Marcelin Berthelot, 75321 Paris Cedex 05, France

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## Reply to Referee 3

*Report: This manuscript provides a very detailed analysis of the non-stabilizerness for two prototypical models of random all-to-all interactions: the full SYK and its quadratic version. The former is known to be more complex and interesting from many points of views, compared to the second and quantum complexity, measured by non-stabilizerness, confirms this expectation. Beside the quantitative benchmarks for the different behavior contained in this work, the only new result seems to be the observation that the Mayorana spectrum for the quadratic model follows a Laplace distribution, thus decaying slower than the Gaussian behavior of the full SYK case. However, this result remains just an empirical observation and no attempt is made to understand it, while it seems rather natural that such behavior should stem from the decomposibility of correlations functions into 2-point functions. But the manuscript does not dwelve in such analysis.*

*Thus, while I fully support the publication of this work, I do not think it meets the high standard of innovation and relevance required by the flagship journal and I recommend it acceptance in Scipost Core.*

Response: We thank the referee for acknowledging that our work provides a detailed analysis of the non-stabilizerness of two models: the SYK model and its quadratic version.

However, we respectfully disagree with the referee's assessment that "the only new result seems to be the observation that the Majorana spectrum for the quadratic model follows a Laplace distribution." The Majorana spectrum of the $SYK_2$ model is indeed one of our new findings, but it is not the only one. Our systematic study of the stabilizer Rényi entropy (SRE) in an interacting fermionic model such as the SYK model—where disorder averaging is required—has not been explored before and is far from a trivial undertaking. Our results further demonstrate that fermionic systems generally exhibit a larger SRE: even the quadratic $SYK_2$ model shows significantly higher magic. From the perspective of stabilizer content, our work indicates that fermionic eigenstates possess intrinsically more magic compared to spin models. Another important aspect concerns the spreading of magic under unitary dynamics, which our work addresses for the maximally chaotic SYK model for the first time.

Regarding the referee's comment on the Laplace distribution of the $SYK_2$ Majorana spectrum, we would like to emphasize that obtaining analytical insight is not *a priori* clear due to the following reasons. Although $SYK_2$ is quadratic and Wick's theorem reduces higher-point correlators to two-point correlators, the SRE involves the evaluation of $4^N$ Majorana expectation values. While each expectation value can be expressed as a determinant of a covariance matrix, computing the full SRE or the Majorana spectrum requires sampling an exponential number of such determinants. Even though efficient algorithms have been proposed in Ref. 41 by M. Collura *et al.* (arXiv:2412.05367) to sample these determinants for Gaussian fermions, obtaining an analytical understanding of the Laplace law for the Majorana spectrum remains far from trivial. Furthermore, since the main focus of our work is the comparison between the SYK and $\text{SYK}_2$ models, we chose to consider system sizes of comparable scale to better highlight the differences in their magic content.

We also note that, even in qubit models, the emergence of a Gaussian distribution is primarily supported by numerical evidence and justified through quantum typicality arguments, rather than derived from analytical predictions. Thus, we do not agree that the result should be regarded as a mere empirical observation. Our findings—both for the ground state and for the steady state under unitary dynamics—indicate that random Gaussian states exhibit a Laplace distribution. We believe this is a generic feature of non-chaotic, integrable random-disorder systems.

*Requested change 1: On page 4, the sentence " On the other hand, odd-parity strings can describe as logical operations." should be corrected for English;*
Response: We thank the referee for pointing out this error. We have corrected the sentence accordingly.

*Requested change 2: After eq. (12): can the author comment if they check that another filling fraction produces similar results or has $N_p = N/2$ been chosen for a particular reason to be clearly stated?*
Response: In the initial stage of our work, we checked that other filling fractions also produce a similar Majorana spectrum, namely Gaussian distribution for SYK and Laplace distribution for SYK2 for few realisation of disorder. Hence, we expect that the same qualitative behaviour for SRE will hold for other filling fractions as well. It is natural to focus on the $N_p = N/2$ sector, since the effects of interactions are strongest at half-filling, and many-body physics in the presence of strong interactions is typically studied in this sector (or the $S^z = 0$ sector in the spin representation).

*Requested change 3: Eqs. (17) and (18) contains delta-function contributions not visible in the plots of Figs . 1: can the authors explain why?*
Response: The observation of the referee is correct. However, in the earlier version of the manuscript, we already mentioned that we have removed the trivial delta-function contribution arising from the expectation values of odd Majorana operators. The corresponding sentence in the manuscript is *'In all the Majorana spectra shown in this work, we have removed trivial zero values of $d^2/2$ expectation values of odd parity Majorana strings, which would otherwise give a delta function peak at $x = 0$. Additionally, we also remove the distinct peak at $x = 1$, arising from the contributions of the identity operator ($\hat{I}$) and the parity operator ($\hat{\mathcal{P}}$)...'*

*Requested change 4 :Does the choice of the initial state in eq. (20) matter for the evolution or other separable, stabilizer states behave similarly?*
Response: The other possible stabilizer initial states are related to our chosen initial state by Clifford operations. Since Clifford operations do not change non-stabilizerness, it follows that, from the perspective of non-stabilizerness, their long-time spectra and SRE will be very similar.

*Requested change 5: I think that a little more details should be given in the conclusion on* *the relation between this work and those in ref. [72,73]*

Response: We thank the referee for this question and address it only insofar as our work overlaps with the cited references.

**Ref. 72.** Ref. 72 studies the SRE and entanglement measures for a combined Majorana SYK and $SYK_2$ setting. Our overlap with Ref. 72 concerns the ground-state SRE: comparing our complex SYK results at system size $N = 14$ with their Majorana SYK results at $N = 28$ (two Majoranas $\equiv$ one complex fermion) suggests that the Majorana SYK model exhibits a slightly smaller SRE than the complex SYK model, though both are of comparable magnitude.

**Ref. 73.** Ref. 73 investigates both unitary and monitored dynamics of the SYK model in the spin representation within the total $S^z = 0$ sector, which corresponds precisely to the half-filling sector of the complex SYK model studied here. Their steady-state SRE under unitary dynamics from a Néel initial state is of the same order as our steady-state SRE obtained from a charge-density-wave initial state.

In our work, we examined both (i) the ground-state SRE (overlapping with Ref. 72) and (ii) the unitary dynamics (overlapping with Ref. 73). In addition, we analyzed the Majorana spectrum, which contains information beyond the moment-based SRE.

We have now incorporated a brief discussion of the overlap with Refs. 72 and 73 into the revised manuscript, included in the *Note Added* section.