Dear Reviewer,
We sincerely thank you for your careful reading of our manuscript and for your positive evaluation.

We have carefully addressed all your comments and suggestions. In the following, we provide detailed responses to each point. For clarity, we highlight the reviewer's comments in blue and our responses in black.

We believe that the revised manuscript has been significantly improved and is now suitable for publication in SciPost Physics Core.

Sincerely,
Michele Viscardi, Marcello Dalmonte, Alioscia Hamma, Emanuele Tirrito

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List of Changes

- line 261: added definition of d and d_A;
- lines 273-286: improved the clarity of the paragraph;
- added footnote 3 and lines 528-529 to specify our bipartition of choice for the spectral quantities mentioned in the manuscript;
- lines 622-632: added a clarification about the magic localisation phenomenon in the quantum XY model;
- updated the references;
- line 383: specified the meaning of d;
- lines 517-526: clarified parameters of the XXZ Hamiltonians and the characteristics of its phase diagram;
- added footnote 4;
- removed Figure 8b.

Answer to Reviewer

> This manuscript presents a comprehensive study of nonstabilizerness in various spin-1/2 quantum spin chains. The authors numerically evaluate the stabilizer Rényi entropy (SRE) and the antiflatness of entanglement spectra to demonstrate their effectiveness in distinguishing different quantum phases and the interplay between entanglement structures and SRE. They also provide, in the introduction, an excellent review of the research field and , in the supplementary material, a summary of the state-of-the-art techniques for the numerical evaluation of SREs.
>
> I believe the manuscript represents a significant contribution to the field and I strongly recommend its publication in SciPost Physics Core, after the implementation of the changes requested below.

We thank the Referee for the positive evaluation of our work and for recommending it for publication.

Equation 20 and line 259: what are d and dA? Are they the dimension of the Hilbert spaces of total system and of A? These quantities should be defined.

We thank the reviewer for spotting this missing definition. We added it immediately after Eq.20 (line 261).

Lines 268-278: this paragraph was hard to follow. Therefore, I would recommend the authors to revise it for improved clarity and flow.

We thank the Reviewer for this helpful suggestion. We have rewritten the paragraph to improve clarity, readability, and logical flow (see lines 273-286).

In Section 4: Which bipartition are the authors considering for the computation of entanglement entropy / antiflatness / capacity of entanglement? I would assume it to be half chain, but it should be specified.

We fully agree with the Reviewer's comment. While in some parts of the manuscript (e.g., Section 4.2) it was explicitly stated that spectral quantities are computed across a half-chain bipartition, this information was missing in other sections. We have now added a clarifying footnote in the opening paragraph of Section 4 and restated our bipartition choice in Section 4.1 to ensure full consistency (see footnote 3 and lines 528-529).

In the analysis of the XY chain in Section 4.2 the authors report a change in the behaviour of nonstabilizerness and antiflatness moving from the regime of small anisotropy to larger one. I appreciate their argument explaining the transition from localised to non-localised magic. However, I keep wondering what is the physical aspect that makes this shift possible? At first I believed that the oscillations could be related to the oscillations in the parity of the ground-state happening below the separability line, but then one would have to observe similar oscillations also for larger values of γ. So I wonder if this is, instead, somehow related to the fact that at $\gamma = 0.01$ we are very close to the other transition line of the model, and hence we observe the change in complexity?

We thank the reviewer for the question. The rationale behind Magic localisation has, indeed, to be traced back to parity oscillations in the ground state of the model at finite sizes, a phenomenon that has been studied in depth in Ref. [1]. As shown in the attached figures, in accordance with Ref. [1], at finite sizes, inside the separability circle, one of the two parity-broken ground states has a lower energy than the other. The parity of the ground state is also found to be oscillating as the parameter h increases in the separability circle. Therefore, when considering the ground state of the full Hamiltonian, we see all its quantum resources to be *oscillating* with the same period of the parity, periodically assuming the values of the even- and odd-sector ground states. As γ increases, however, the parity oscillations get smoother and smoother, as for the oscillations in our quantities of interest, which explains why we see strong oscillations only for low values of γ, near the isotropic limit of the model. We updated the manuscript accordingly with the Reviewer's concern to provide a better explanation to this magic localisation phenomenon (see lines 622-632).

References

[1] Antonella De Pasquale and Paolo Facchi. *xy* model on the circle: Diagonalization, spectrum, and forerunners of the quantum phase transition. *Phys. Rev. A*, 80:032102, Sep 2009.

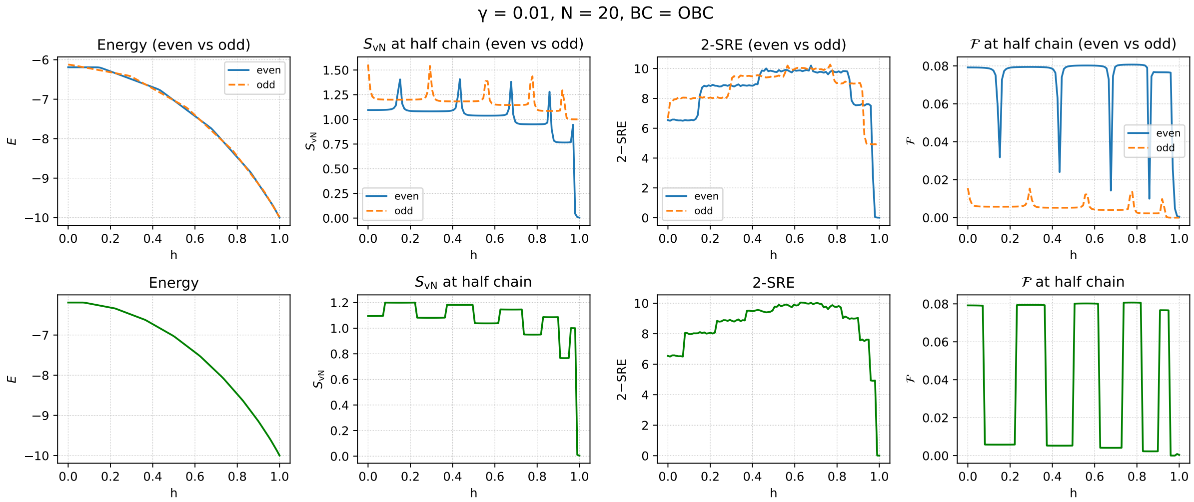

Figure 1: Top row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the ground states of the even (blue, solid line) and odd (orange, dashed line) parity sectors. Bottom row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the parity-broken ground state with minimum energy.

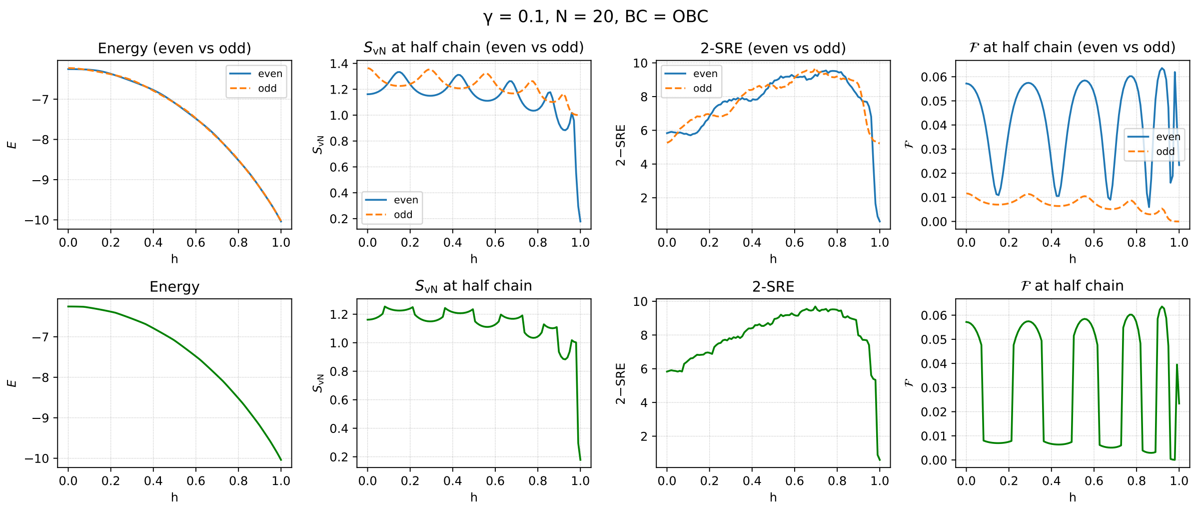

Figure 2: Top row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the ground states of the even (blue, solid line) and odd (orange, dashed line) parity sectors. Bottom row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the parity-broken ground state with minimum energy.

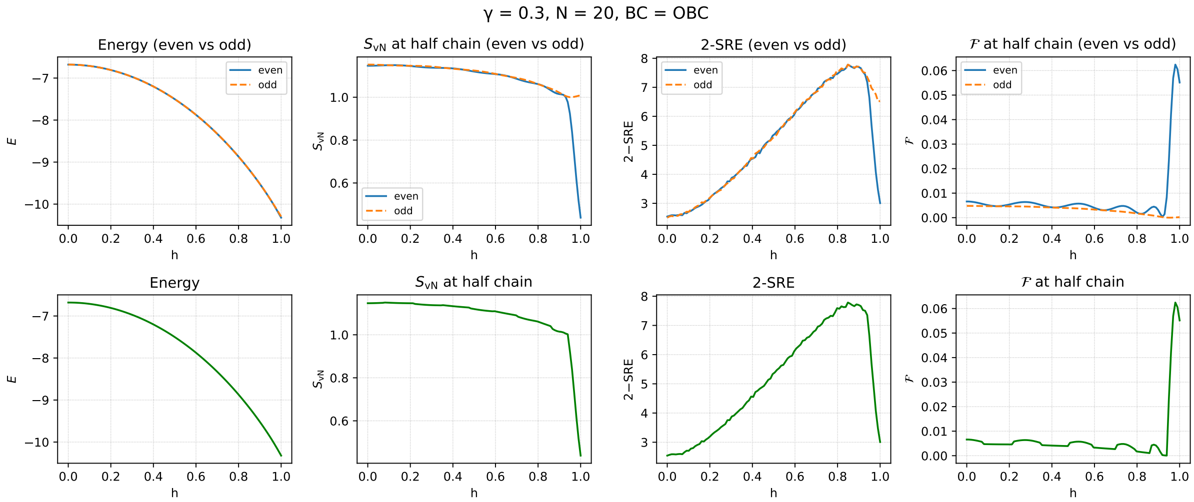

Figure 3: Top row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the ground states of the even (blue, solid line) and odd (orange, dashed line) parity sectors. Bottom row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the parity-broken ground state with minimum energy.

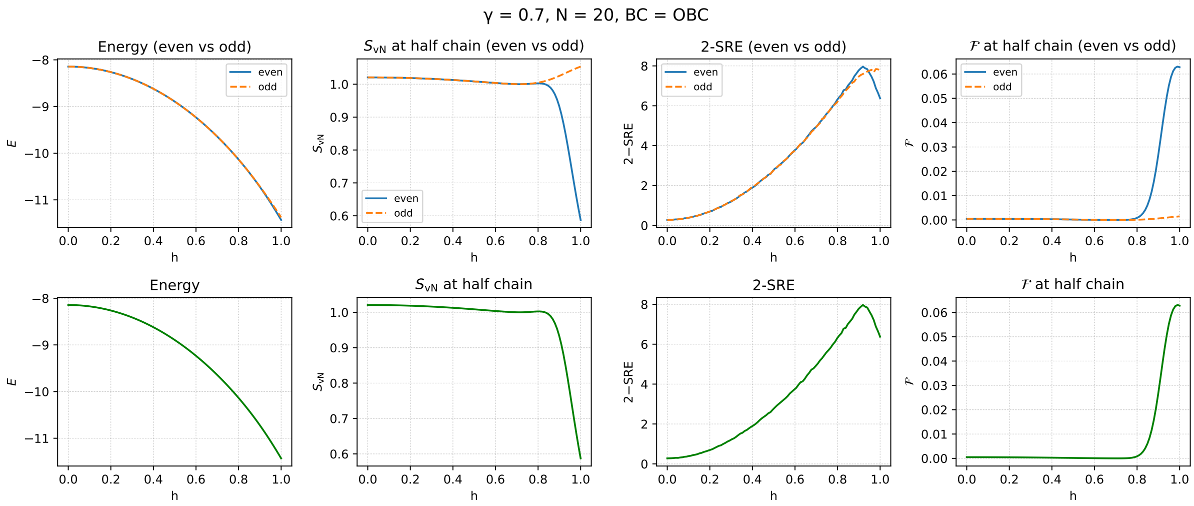

Figure 4: Top row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the ground states of the even (blue, solid line) and odd (orange, dashed line) parity sectors. Bottom row: energy (E), Von Neumann entropy ($S_{\rm vN}$), 2-SRE and Antiflatness ($\mathcal{F}$) of the parity-broken ground state with minimum energy.