In what follows we list the text added to the manuscript.

On the comparison with the gluon. We have added the following text: (1) at the beginning of Section 2.2 to clarify quark couplings to Θ; (2) near Eq. (3.3) discussing existing bounds on Θ; (3) at the beginning of Section 2.2.2 on the decoupling of Θ in the large mass limit.

1. Note that although the scalar studied here and the gluon are both color-octet electroweak-singlet bosons, their couplings to quarks are fundamentally different: the spin 1 of the gluon allows it to have a large coupling to quarks, while the spin 0 of Θ implies that the interplay between the gauge and Lorentz symmetries forbids renormalizable couplings to quarks.

2. Comparing the CMS limit [8] on $\sigma\, B_{\rm tot}\, A_{4j}$ with the $4j$ signal discussed here, we find a lower mass limit on Θ of about 0.9 TeV. This is only an estimated limit, as it is based on the CMS limit for squarks; we will show in Section 4 that the distribution shapes for Θ and squarks are somewhat different.

3. All experimental constraints on Θ go away in the decoupling limit where the Θ mass is very large. We will see in Section 3 that the current lower limit on M_Θ is set by the CMS search for $(q\bar{q})(q\bar{q})$ at about 0.9 TeV. This is low enough to warrant the question of what other channels sensitive to Θ may be probed at the LHC.

On constraints from other processes: We have expanded on this point by creating a new subsection 2.2.2 titled "Other phenomenological implications of the Θ couplings to quarks". In addition to the point of mass decoupling written above, the new text includes (1) a discussion on the dijet final state, and (2) dijet+SM boson final state.

1. Before discussing some potentially promising channels, let us comment on the single production of Θ from a quark-antiquark initial state, which could be targeted by dijet resonance searches [1, 2]. Using Madgraph [3], we find that the LO cross section for single production of a 0.9 TeV color-octet real scalar is $\sigma(pp \to \Theta) \approx (41\ {\rm pb})\, y_{\Theta u}^2$ where $y_{\Theta u}$ is the effective Yukawa coupling of Θ to the up quark, shown in the second term of (2.12); the $i = j = 1$ indices are not shown here. The most stringent limit at that mass was set by CMS [1] at approximately $\sigma(pp \to \Theta) < 0.5$ pb, so the upper limit on the coupling is $y_{\Theta u} < 0.11$. This can be translated into a lower limit on the mass

scale that suppresses the corresponding higher-dimensional operator (2.13): $M_*/C_{\Theta u} > 1.6$ TeV, which is a weak constraint given that M_* is expected to be larger than M_Θ by a factor of about two or larger, while $C_{\Theta u}$ is at most of order one.

2. Single-Θ production followed by the decay into a $q\overline{q}$ pair plus a boosted SM boson would appear as a trijet resonance when the boson decays hadronically. A search for this topology has been performed only for resonance masses above 1.75 TeV [4]. The leptonic decay of the SM boson, in association with the two jets from the Θ decay, would suffer from smaller backgrounds, but the rate for this type of process is low. For some typical values of the parameters, $M_*/C_{\Theta u} \approx 5$ TeV, the effective Yukawa coupling is $y_{\Theta u} \approx 0.035$, giving a cross section, $\sigma(pp \to \Theta) \approx 50$ fb, smaller than the one for pair production. Since the branching fractions for the 3-body Θ decays are at most of order 1%, searches for these signals will require the larger data sets of the future High-Luminosity LHC runs.

On the statistical interpretation. In Section 4, we have added: (1) a discussion of a performed hypothesis test, and (2) a discussion on the assumptions that this evidence relies on.

1. To quantify the compatibility of the data with the $\Theta_{\rm C}$ hypothesis relative to the background-only hypothesis, we construct a test statistic as the log likelihood ratio between the two hypotheses and perform a hypothesis test [5, 6, 7]. Taking the asymptotic limit, we calculate the local p-value to be 1.4×10^{-3} corresponding to 3.0σ. Such a strong preference for the inclusion of the signal is remarkable given that the mass is the only free parameter in the signal model. Note that we have not even optimized the fitted mass, instead taking the best-fit obtained by CMS for squarks. The fit of $\Theta_{\rm C}$ also improves overall compared to the octet real scalar, $\chi^2_{\Theta+{\rm B}} = (24.1, 30.5, 7.7)$ corresponding to a 2.6σ preference, due to the smaller χ^2 in the central bin, even though the fit is not as good in the side $\overline{M}_{jj}/M_{4j}$ bins.

2. The above evidence depends on the choice of background model (*i.e.* ModDijet-3p, see Section 3) and therefore more accurate predictions of high-$p_{\rm T}$ 4-jet production in the SM are warranted to reduce the uncertainties associated with this prior. Furthermore, the evidence depends on theoretical uncertainties related to both the signal cross section and shape. The former are associated with the renormalization and

factorization scales used in the NLO computation of the cross section (17% at NLO, see Section 3), and with the parton distribution (of order 10%). The latter is due to differences in the kinematic distribution for the signal, which we simulate only at LO.

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