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I. WHAT HAS BEEN STUDIED IN THIS PAPER ?

This is a very interesting work where the physics of small systems and their hydrodynamic behaviour has been discussed in the context of Boltzmann-Vlasov equation.

In this work, the author has discussed the onset of hydrodynamic behaviour of small systems by attributing such a phenomena to fluctuations in the system that enter in to the distribution function $f(x,p)$. In small systems, the distribution function is not known and should be guessed. According to this work, the change in the distribution function can be modelled as a *Gibbsian*, through Bayesian analysis. The starting point of this study is the Boltzmann equation, in which an extra term, known as the Vlasov term has been introduced, thus promoting the equation to Boltzmann-Vlasov equation. The features / purpose of this Vlasov term is the following :

- The introduction of the Vlasov term ($\mathcal{V}^\mu$) is justified because for *small-systems which are highly correlated*, the instabilities in the system require a term which is of $\mathcal{V}^\mu$ type.
- The long-range interactions are taken care of through the $\mathcal{V}^\mu$ term in the Boltzmann-Vlasov equation. The collision term ($C[f]$) in the Boltzmann equation takes care of the quantities calculated from the scattering elements ($|\mathcal{M}|^2$) viz. cross sections, decay widths etc.

The distribution function has been represented as a functional by expressing it as a probability density in the form of a Gaussian ansatz. This has been obtained via technique of *Wigner functions*. The assumed Gaussian ansatz for the probability density of distribution function, carries a functional width $\sigma_f \sim \sqrt{N_{\mathrm{DoF}}}$, which dictates how the probability density for the distribution function varies with the degrees of freedom of the system. A gauge redundancy has been claimed to exist even in the limit of narrow σ_f and is responsible for the fluidic behaviour of small systems.

II. QUESTIONS / COMMENTS

1. In Eq. (2), the assumption of the distribution function is not very clear. A sketch of some calculations showing how Eq. (2) is derived, would be helpful.
2. It has been argued that the introduction of the *Vlasov term* and further using a Gibbsian estimation of the distribution function, induces a *gauge redundancy* in the system.
   Does the Vlasov term act like a background gauge field in the system ? A comparision made with the Boltzmann-Equation in the presence of background magnetic field shows that a force term coming from magnetic field looks similar to the $\mathcal{V}^\mu$ term in this paper.
   It is requested to look into :
   (1) Ref - G.S. Denicol *et.al* [1]
   (2) Ref - A. Dash *et.al* [2]
3. In Pg - 7, the following claim has been made : *" The set of transformations leaving the RHS of eq. 4 invariant can be thought of as defining a "Wilsonian" flow across the space of f (...) which are part of the same Gibbsian ensemble."*. Some explanation is required, particularly in the UV and IR regimes that how the RHS of Eq. (4) remains invariant under such transformations. Is the set of transformations that leaves Eq. (4) invariant related to the *scale transformations and local scale transformations* encountered in RG flow ? It would be useful if some explanation can be offered in this regard.
4. It has been argued in Sec. 2 of Appendix, that for a *strongly correlated system* the BBGKY hierarchy cannot be used as an expansion, rather a non-perturbative technique be used for this purpose. But a non-perturbative method such as *Mean Field Theory* might be used for this purpose -
   (1) Ref - M.B. Pinto [3]
   (2) Ref - P. Romatschke [4]
   This might be looked into for studying a strongly coupled system and some explanations are required for this in the manuscript. Please include these references in the manuscript which may offer scope for interesting studies in the future.

5. There are some typos in the manuscript which need to be corrected.

[1] G. S. Denicol, E. Molnár, H. Niemi and D. H. Rischke, "Resistive dissipative magnetohydrodynamics from the Boltzmann-Vlasov equation," Phys. Rev. D **99**, no.5, 056017 (2019) [arXiv:1902.01699 [nucl-th]].
[2] A. Dash, S. Samanta, J. Dey, U. Gangopadhyaya, S. Ghosh and V. Roy, "Anisotropic transport properties of a hadron resonance gas in a magnetic field," Phys. Rev. D **102**, no.1, 016016 (2020) [arXiv:2002.08781 [nucl-th]].
[3] M. B. Pinto, "Three dimensional Yukawa models and CFTs at strong and weak couplings," Phys. Rev. D **102**, no.6, 065005 (2020) [arXiv:2007.03784 [hep-th]].
[4] P. Romatschke, "Quantum Field Theory in Large N Wonderland: Three Lectures," [arXiv:2310.00048 [hep-th]].