# Report on the Lecture Notes *"Les Houches Lecture Notes on Topological Recursion"* by Vincent Bouchard

These lecture notes provide a clear, modern, and conceptually coherent introduction to topological recursion, with a particularly successful emphasis on Airy structures and differential constraints as an organizing principle.

## Positive aspects

A major strength of these lecture notes is the clarity of their conceptual organization and the choice of perspective adopted throughout the text. Rather than introducing topological recursion directly in its original Eynard–Orantin residue formulation, the author begins with Airy structures and differential constraints. This modern point of view provides a unifying algebraic framework that naturally encompasses Virasoro constraints, $W$-constraints, and topological recursion itself. As a result, the reader is guided toward an understanding of topological recursion as a structural phenomenon, rather than as a collection of ad hoc recursive formulas.

The exposition is particularly successful in explaining why Airy ideals represent a notion of "maximal" systems of differential constraints and how such systems lead to uniquely determined partition functions. The formulation in terms of the Rees Weyl algebra and the Bernstein filtration is handled with care, and the motivation for each definition is clearly explained. The central existence and uniqueness result for Airy partition functions (Theorem 2.11) is presented at the right level for lecture notes: the statement is precise, while the proof is outlined in a way that conveys the key ideas without overwhelming the reader with technical details.

From a pedagogical standpoint, the notes are very well written. The author consistently takes the time to explain why certain constructions are introduced and how they relate to classical objects such as the Kontsevich–Witten tau-function, the BGW model, and Virasoro representations.

Another strong point is the breadth of connections highlighted throughout the notes. The author succeeds in conveying how topological recursion sits at the intersection of integrable systems, enumerative geometry, representation theory, and mathematical physics.

The choice of examples is particularly effective. The Kontsevich–Witten and BGW Virasoro constraints are presented not merely as classical results, but as prototypical instances of the general Airy framework.

Finally, the notes reflect the current state of research in the field. The references are up to date, and several recent developments are mentioned, including generalized Airy structures, new cohomological classes on moduli spaces, and relations with quantum curves and resurgence. This makes the notes not only a pedagogical introduction but also a useful gateway to contemporary research directions. Overall, the lecture notes combine clarity, depth, and modern perspective in a way that makes them a valuable resource for both newcomers and active researchers.

## Misprints and points for clarification

- I highly recommend adding a table of contents.
- p. 4 (around Eq. (2.2)): *"intepretation"* should read *"interpretation"*.
- p. 4: *"particulat tau-function"* should read *"particular tau-function"*.
- p. 7, Definition 2.7: *"partition fuction"* should read *"partition function"*.
- p. 6: *"Our main of object of study"* should read *"Our main object of study"*.
- p. 11. after Equation 2.28: *Of course, Z is nothing but th KW tau functions.* Maybe it would be instructive to add the link or some explanation.
- p. 23, Remark 3.14: the phrase *"which is a common way of writing in the literature"* is duplicated.
- p. 30, Equation 3.56: $dz$ is missed.

## Summary

These lecture notes are of high quality and provide a clear and modern introduction to topological recursion and Airy structures. I recommend the paper for the publication after correcting minor typographical issues.