Referee report for SciPost Physics Core
Thermodynamic integration, fermion sign problem, and real-space renormalization
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This paper uses Monte Carlo calculations to study a real space RG map for 2d Ising type models related to what Wilson did in his 1975 Rev Mod Phys paper. It would be very interesting to reproduce and extend the results in Wilson's paper since (as far as I know) no one has ever reproduced these results in which Wilson studies the RG maps with a very large number of terms in the Hamiltonian. However, that is not accomplished in this paper.

To do Monte Carlo for a real space RG with the RG map they consider one must first consider

1. thermodynamic integration - how do you compute quantities like free energy in a MC simulation

2. the sign problem introduced by their choice of RG map

3. you need to take derivative (more precisely numerical approximation thereof) and this is tricky for quantities that are computed by MC.

1 is a standard problem in MC studies of statistical mechanics model

The sign problem 2 is really only a problem when the average value of the sign is close to zero. They do some numerical studies that show it is not close to zero. And so they can handle the mild sign problem with standard methods [12].

For 3 they do some numerical studies which show that correlated sampling can overcome this problem.

Steps 1, 2 and 3 are the first steps that must be taken before a MC study of the RG map can be carried out. But they do not go beyond these initial steps in any significant way. They have an approximate calculation of the fixed point when you only include NN and NNN terms in the Hamiltonian, i.e., 2 factor types. Almost all of the computations in this paper are for the 2 factor case. For the 14 factor model it appears that the only calculation that has been done is to check that the sign problem is not an impediment.

Wilson included 18 factor types in his studies which found a fixed point, computed critical exponents and studied the flow of the RG map. This work does not compute any critical exponents or study the flow of the RG map. In the abstract they write that they "retrace Wilson's computation for a real-space renormalization with a number of terms in the hamiltonian." This is a gross misrepresentation of what is actually done in this paper.

There has been a vast amount of work on using MC to study real space RG maps, but this paper does not discuss this work or its relation to the method in this paper at all. (There are zero references to other MC RG work.)

The acceptance criteria for the journal require that the paper

- Address an important (set of) problem(s) in the field using appropriate methods with an above-the-norm degree of originality
- Detail one or more new research results significantly advancing current knowledge and understanding of the field.

This paper certainly addresses an important problem, and their computations show that it may be feasible to use their MC methods to study the RG map. But this paper does not contain any results that signicantly advance the current knowledge and understanding of RG maps. So I recommend that the paper be rejected.