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Title: Twisting asymptotically-flat spacetimes
Authors: Marc Geiller, Pujian Mao, and Antoine Vincenti
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The paper discusses in detail a novel relaxation of Bondi coordinates to describe four-dimensional asymptotically flat spacetimes as solutions of Einstein gravity. This relaxation amounts to choosing r as a null coordinate along outgoing null geodesics which are allowed to possess a non-trivial twist (in contrast with the standard Bondi–Sachs case), u as a retarded time, and x^a as angular coordinates on the celestial sphere. The coordinates introduced by the authors conveniently allow for remarkable resummation properties (in terms of a series expansion in $1/r$) of algebraically special solution spaces, in particular for the paradigmatic example of Robinson–Trautman waves, yielding a simpler metric on the sphere (typically the unit round-sphere metric). From the boundary perspective, the presence of a non-trivial twist makes it possible to introduce a non-trivial Ehresmann connection on the conformal Carroll structure at null infinity. This is an important geometrical ingredient, necessary both to discuss invariance properties of boundary theories under local Carroll transformations and to define a boundary connection, which is crucial for describing bulk gravitational radiation from a boundary viewpoint (following Ashtekar and more recent developments in the Carrollian literature).

Overall, the analysis is well written, pleasant to follow, and covers all the aspects one would expect from such a construction. The authors first provide a complete description of the coordinate choice and of the solution space, both in the convenient Newman–Penrose formalism and in a more direct metric formulation. They then derive the residual gauge symmetries and compute the associated charges, thereby promoting these transformations to genuine asymptotic symmetries. The resulting asymptotic symmetry group consists of Carrollian diffeomorphisms (*i.e.* boundary diffeomorphisms preserving the adapted coordinate system, with ξ^u depending on all boundary coordinates while ξ^a is u-independent) together with local Carrollian boosts Υ^a. Finally, they particularise to the aforementioned class of solutions and test their construction against the particularly tractable example of three-dimensional gravity. The topic is timely, the results appear scientifically sound and the important technical effort should be praised. I would therefore be very happy to recommend this paper for publication in *SciPost*, provided the authors address the minor comments listed below.

1. In Section 4, the boundary gauge is fixed in such a way that boundary Weyl transformations W are locked to Carrollian diffeomorphisms, as expressed in Eq. (4.6a). However, Eqs. (4.9) and (4.10) appear to treat W as an independent parameter, without a clear explanation. The authors should be fully explicit about the boundary gauge fixing and the counting of non-trivial asymptotic symmetries, and contrast their discussion with, for instance, Refs. [54, 55], where the symmetry group is larger and includes non-trivial leading and subleading Weyl symmetries. At the end of the paragraph, I particularly appreciated the sentence "This is a typical example. . . " and believe it would benefit from being completed by the (in my opinion too rarely stated) remark that the paradigmatic example of this phenomenon is essentially BMS_4.

2. In the computation of the charges, Eq. (4.11) appears to be an explicit expression of the standard Iyer–Wald charge, involving only two terms: the first corresponding to the Noether–Wald charge, and the second to the presymplectic potential contracted with the evolutionary vector induced by the AKV. Are the authors certain that this charge indeed descends (or rather ascends) from the contraction of a canonical presymplectic current in the present case and with their choosen loose boundary conditions, and if so, which one (Einstein–Hilbert? renormalised?)? In particular, when dealing with field-dependent gauge parameters, the Noether–Wald charge evaluated on the

variation $\delta\xi$ may contribute to the charge formula. A brief discussion of this subtlety would be very valuable.

3. Regarding Eq. (4.12), the authors correctly note that the parameter Υ^a reduces to the gradient of f in the Bondi–Sachs gauge fixing. What is the resulting expression of this charge in the standard case? My impression is that only the first integrable term survives. Up to an integration by parts on the sphere, I can recognise the contribution $f D_a D_b C^{ab}$ to the energy flux. Could the authors make this explicit and precise and comment on how this affect the BMS charges commonly adopted in the literature (*i.e.* $\sim 4f \,\mathrm{Re}\,\Psi_2^0 + 2Y\Psi_1^0 + 2\bar{Y}\bar{\Psi}_1^0$)?

4. Closely related to the previous point, it seems somewhat strong to state that Carroll boosts are broken by the Bondi–Sachs gauge fixing. Rather, they are not completely broken but reduced, in the sense that their gauge parameter becomes the gradient of an arbitrary function on null infinity, provided by the vertical component of boundary diffeomorphisms. In this way, the boundary Ehresmann connection remains stable under the combined action of local Carroll boosts and diffeomorphisms. In particular, when considering the boundary variational principle in an AdS/CFT-like fashion and its symmetries, invariance under local Carroll boosts must still be demanded, see Ref. [2505.00077] or Ref. [139].

5. From the perspective of constructing coordinate systems that accommodate the full geometric freedom of the conformal Carroll structure at null infinity, Petropoulos and collaborators have developed a coordinate system based on the Newman–Unti gauge, in which the boundary Ehresmann connection is left arbitrary. See for instance Ref. [138], where this coordinate system is clearly presented, which evidences that the authors are already aware of these works. Although a complete analysis of asymptotic symmetries and charges was not carried out there, it would be interesting for the authors to comment a bit further on the relationship between that framework and the present analysis, as well as on the improvements brought by the latter.

6. Importantly, the critical analysis of Ref. [2505.00077] should be cited alongside the preprint [139], and everywhere the latter is referenced. Although both papers address the same issue — namely, the derivation of gravitational flux-balance laws from a boundary perspective — Ref. [2505.00077] appeared earlier, provides a clearer discussion of the boundary structure and of the gauge fixing leading to the usual Bondi–Sachs gauge, and employs a boundary first-order formalism that is readily translatable into Newman–Penrose variables, as used in the present work.

7. A few minor typographical and stylistic remarks:

   (a) Penultimate paragraph of Section 1: *superroation* $\rightarrow$ *superrotation*;

   (b) First sentence of page 11: *appart* $\rightarrow$ *apart*;

   (c) Some hyphenations should be harmonised, for instance *time-dependency* versus *time dependency*;

   (d) Frequent use of expressions such as "in particular", "importantly", and "this can be done by", which could be replaced by suitable synonyms to further improve the flow;

   (e) In the first sentence of Section 3, I think that the correct expression is *"deceptively simple"*.