1 A counterexample for the gauge field case ?

Let us write the wavefunctional for the electromagnetic case. Before the gauge coupling is introduced, we can take the matter wavefunctional as

$$\Phi_0[\phi_1(x), \phi_2(x)] \tag{1}$$

where ϕ_1, ϕ_2 are real fields.

The Lagrangian is

$$L = -\frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\partial_a\phi_1\partial^a\phi_1 - \frac{1}{2}\partial_a\phi_2\partial^a\phi_2 + qA_aj^a \tag{2}$$

where

$$j_a = \phi_1\partial_a\phi_2 - \phi_2\partial_a\phi_1 \tag{3}$$

We will take a gauge where

$$\vec{\nabla}\cdot\vec{A} = 0 \tag{4}$$

The t component of the gauge field equation gives the constraint

$$F_{0i,i} = j_0 \tag{5}$$

where

$$F_{0i} = A_{i,0} - A_{0,i}, \quad F_{0i,i} = A_{i,0i} - A_{0,ii} = (\vec{\nabla}\cdot\vec{A})^{\cdot} + \triangle\Phi \tag{6}$$

In the gauge (4) this becomes

$$\triangle\Phi = qj_0 = q(\phi_1\partial_0\phi_2 - \phi_2\partial_0\phi_1) \tag{7}$$

(Note that this is not the constraint equation in a more general gauge. This will be relevant because sometimes apparently nonlocal effects can arise because of a gauge choice, but there is really no nonlocality because the observables must also be defined carefully in each gauge.)

There are many ways to pass to the quantum theory. One possibility is to impose (7) as an operator relation

$$\triangle\Phi(x)\Psi[\phi_1(x),\phi_2(x),\Phi(x)] = q\left(\phi_1(y)(-i\frac{\partial}{\partial\phi_2(y)}) - \phi_2(y)(-i\frac{\partial}{\partial\phi_1(y)})\right)\Psi[\phi_1(x),\phi_2(x),\Phi(x)] \tag{8}$$

where we have written

$$\Phi = -A_0 \tag{9}$$

Our goal is to see if we can get a solution to the quantum wavefunctional that (i) satisfies the constraints (ii) changes can be made to this wavefunctional at $r < R$ while making no change outside $r = R$; we will argue this means that the state inside $r = R$ cannot be detected from $r > R$ by any means.

The wavefunctional Ψ depends on $\phi_1(x), \phi_2(x), \Phi(x)$. To construct a wavefunctional satisfying the constraints, We proceed in the following steps:

(i) First choose any one function $\Phi(x) = \Phi_1(x)$. Compute

$$U(x) \equiv \triangle\Phi_1(x) \tag{10}$$

which is just a function over 3-dimensional space.

(ii) Now look at any point $x = x_1$. At this point we have a number

$$U_1 \equiv \triangle\Phi_1(x)\Big|_{x=x_1} \tag{11}$$

At this point in function space $\Phi = \Phi_1(x)$, focus on the spatial point $x = x_1$. Here the functional Ψ has to satisfy the constraint

$$\left(\phi_1(y)(-i\frac{\partial}{\partial\phi_2(y)}) - \phi_2(y)(-i\frac{\partial}{\partial\phi_1(y)})\right)\Psi[\phi_1(x_1), \phi_2(x_1)] = \frac{U_1}{q}\Psi[\phi_1(x_1), \phi_2(x_1)] \tag{12}$$

Now this is an equation for a function Ψ of just two real number arguments $\phi_1(x_1), \phi_2(x_2)$. There are many solutions of this equation. Let us call them

$$[\phi_i(x_1)]_1, \quad [\phi_i(x_1)]_2, \quad [\phi_i(x_1)]_3 \dots \tag{13}$$

for later use. (Here $i = 1, 2$ ranges over the two flavors of scalar fields that we have taken.) Note that this equation does not involve the functional Ψ at any other space point $x \neq x_1$ right now. For any other point x_2 we will get similar solutions

$$[\phi_i(x_2)]_1, \quad [\phi_i(x_2)]_2, \quad [\phi_i(x_2)]_3 \dots \tag{14}$$

and so on, where in (12) we replace U_1 by its value U_2 at the point x_2. Thus we can choose a solution to this equation at each point x separately. Suppose we choose the first solution $[\phi_i(x)]_1$ for each x. This gives Ψ that satisfies the constraint everywhere for the point in function space $\Phi = \Phi_1(x)$.

(iii) Now choose some other point $\Phi = \Phi_2(x)$ in the space of functions $\Phi(x)$. Proceed as above, getting a solution that satisfies the constraint at this point $\Phi_2(x)$ in function space. Doing this for all functions $\Phi(x)$ gives us a complete functional $\Psi[\phi_1(x), \phi_2(x), \Phi_0(x)]$ that satisfies the constraint.

(iv) Now we observe that since for each $\Phi(x)$ the construction above proceeded separately for each point x, we can choose two different functionals $\Psi[\phi_1(x), \phi_2(x), \Phi(x)]$ that are the same for all points outside a sphere $r = R$ and differ inside. Then we will not be able to do any observations outside $r = R$ to find what is the state inside.

(v) Note that the above state was a factored product of states at each x. This is not a generic low energy state. But we can take a state that has some entanglement between nearby points, by taking superpositions of the above constructed Ψ; thus we can for example add the Ψ that we get from choosing $[\phi_i(x)]_1$ at each x to the wavefunctional that we get from choosing $[\phi_i(x)]_2$ at each x. This is a more general class of states, but again, we can arrange that the wavefunctional outside $r = R$ remain unchanged while the wavefunctional inside $r = R$ is altered.

2 The gravity case

At the formal level at which one is working, the gravity set up looks similar to this electromagnetic set up. The gauge potential $A_0(x)$ is replaced by $h_{00}(x)$, and the current is replaced by the stress tensor $j_0 \to T_{00}$ where

$$T_{00} = (\nabla\phi_1)^2 + (\nabla\phi_2)^2 + \Pi_1^2 + \Pi_2^2 \tag{15}$$

At the quantum level we can write

$$\hat{T}_{00} = (\nabla\hat{\phi}_1)^2 + (\nabla\hat{\phi}_2)^2 + \hat{\Pi}_1^2 + \hat{\Pi}_2^2 \tag{16}$$

which becomes, in the representation $\Psi[\phi_1(x), \phi_2(x), h_{00}(x)]$

$$\hat{T}_{00} = (\nabla\phi_1)^2 + (\nabla\phi_2)^2 - \frac{\partial^2}{\partial\phi_1^2} - \frac{\partial^2}{\partial\phi_2^2} \tag{17}$$

The analogue of (**??**) becomes

$$\triangle h_{00}(x) = \left((\nabla\phi_1(y))^2 + (\nabla\phi_2(y))^2 - \frac{\partial^2}{\partial\phi_1(y)^2} - \frac{\partial^2}{\partial\phi_2(y)^2} \right) G[\phi_1, \phi_2] \tag{18}$$

It is not clear to me that a construction similar to the electromagnetic case will not work, but if checking this will be relevant, then I will be happy to try.