Referee report on "Les Houches lecture notes on moduli spaces of Riemann surfaces", by A. Giacchetto and D. Lewanśki

Review

These lecture notes aim to be a gentle, but on the whole systematic introduction to moduli spaces of stable curves and their relation with integrability and other topics in mathematical physics. The subject matter is covered from the ground-up with starting graduate students in theoretical physics as the main target audience. The authors review moduli of Riemann surfaces, their Deligne–Mumford compactification and boundary structure thereof, Witten's conjecture and connection to integrability in the form of Virasoro constraints, and then move on to more advanced subjects with a description of Kontsevich–Manin CohFTs, R-matrix theory, and the Givental–Teleman classification in the semi-simple case. Some pointers outlooking further topics of active interest, from JT gravity to topological strings and Gromov–Witten theory, are then provided in the final section.

Assessment

This is a really excellent short introduction to the subject of moduli of curves and their inroads into mathematical physics. It would doubtless be hard to do full justice to this subject within the span of less than 50 pages; I've got to say the authors have done a pretty great job in giving a primer to the uninitiated which is at the same time self-contained – with little-to-no background previously assumed, and in particular no exposure to algebraic geometry really assumed anywhere, the language being largely topological or differential-geometric in character throughout – and relatively extensive, ramping up speed quickly (but sensibly) to cover active research-level topics. The choice of exercises is spot on and instructive at every step, giving a good balance between laying the basics as effectively as possible for a broad audience, yet throwing a bone to more ambitious students with progressively non-trivial questions at appropriate points of the text. The progression to more challenging topics is gradual throughout, yet succeeds getting to point of tapping to research-level questions in the two later sections of the Lectures. The writing style is particularly worthy of commendation – neat, to-the-point, and well-targeted pedagogically at the desired audience (while not stated, this is implicitly given by starting hep-th PhDs; which is a welcome step given that the authors are both pure mathematicians). I've got to say that I had to struggle to find any relevant bugs to be fixed in the current text, a list of which is given below. On the whole, all suggestions are minor, and I would recommend publication if they were implemented.

Minor requests for revision

1. Line 80: you may want to add "without boundary" after "compact".

2. Line 125: "after glueing": for the reader's convenience, you may wish to be a little more specific about what you're glueing and how.

3. Line 129: "as as" should be "as"

4. Line 149: you really mean "smooth as a DM stack" here, but given the natural target audience of these lecture notes, there may be some surprise in seeing "smooth" in the same sentence where quotient singularities are allowed. You can try a paraphrase such as "smooth other than for (finite order) quotient singularities".

5. Line 161: I wonder if throwing in a hint with taking $d \gg 1$ so as to invoke Kodaira vanishing might be worth – I'd leave this one to the authors' taste really.

6. Line 178: "integral" should be "integrals".

7. Line 179: add spacing before "The non-compactness".

8. Line 196: nit-picking here but you're saying "singular" and "manifold" in the same sentence. Maybe replace "1-dimensional manifold" with "curve" to be in the clear?

9. Line 247: "rational" should be "rational curve"

10. Line 290: you may wish to spell out explicitly that your notation $e = (h, h')$ stands for the two half-edges making up the edge e.

11. Line 305: "coincide" should be "coincidence"

12. Line 307: "maps" should be "map"

13. Line 351: "Chern classes of natural complex vector bundles over" ...what? (just drop "over").

14. Line 422, eq. (2.54): at the risk of being pedantic, do you want to explain your notation for the marked points/leaf markings in your stable graph picture – something like the customary "where as usual, a caret as in $\widehat{i}$ denotes omission".

15. Line 430, eq. (2.56): $\psi_i^{d_i}$ should be $\psi_j^{d_j}$.

16. Line 612: "know" should be "known"

17. Line 658: it sounds unnecessary, and certainly cruel, to refer a physics-minded reader to [45], where they would need to trawl their way to the construction of suitable morphisms to the truncated cotangent complex on the moduli space of stable maps, but where in all likelihood the expression "topological string amplitudes" is nowhere to be found. One could perhaps refer to Kontsevich–Manin here, which is orders-of-magnitude closer to the spirit of these lecture notes (although obviously pre-dating the construction of the GW vfc in algebraic geometry).

18. Line 799: I believe the proof of the statement in the exercise is due to [51], and in fact to its predecessor preprint `1104.0176`, which perhaps deserve being referenced more than [30]-[32] and [53] in this context.