Referee Report on "Topology-induced symmetry breaking demonstrated in antiferromagnetic magnons on a Möbius strip" by Deng and Cheng

In this work, the authors explore the influence of real-space topology, specifically, that of the Möbius strip, on the eigenmodes (magnons). By considering a model composed of two ferromagnetic spin chains coupled antiferromagnetically and arranged into a Möbius strip, they demonstrate that topological boundary conditions (TBCs) can induce a form of symmetry breaking in the excitation spectrum, which they name **topology-induced symmetry breaking (TISB)**.

The key result is that despite the Hamiltonian preserving local O(2) spin rotational symmetry, the Möbius topology forces all magnon eigenmodes to be **linearly polarized**, eliminating chirality and leading to a split into two non-degenerate branches with distinctive standing wave patterns (one supporting half-integer modes, the other integer modes). This effect persists even in the absence of curvature and is fundamentally distinct from spontaneous symmetry breaking of the ground state.

The concept of *topology-induced symmetry breaking* is, to the best of my knowledge, novel and presents a new way of thinking about how boundary conditions can modify the symmetry of excitations.

Importantly, the work goes beyond superficial analogies and develops a fully microscopic model (Heisenberg Hamiltonian), applying analytical methods to obtain and interpret the magnon modes. The idea that Möbius topology alone can eliminate chirality from spin wave excitations, even when chirality would otherwise be protected by local symmetry, is both original and of general interest. The results are potentially applicable to engineered meta-materials (e.g. magnonic crystals).

The term "topology-induced symmetry breaking" (TISB) is compelling but perhaps deserves a clearer definition upfront, with a contrast to spontaneous symmetry breaking in the introduction.

The authors correctly handle the antiferromagnetic spin wave theory using Holstein–Primakoff and Bogoliubov transformations, carefully adapting them to Möbius TBCs. They identify that the Möbius boundary condition mixes sublattices and prevents the eigenmodes from being globally circularly polarized.

The "exotic phase" with **ferromagnetic interchain couplings** but preserved AFM order is a particularly intriguing prediction. The analysis of the soft mode and stability threshold is convincing. Could there be **experimental observables** that directly detect the spectral shift or suppressed chirality, beyond the proposed time-resolved microscopy? A brief discussion of **fermionic systems**, such as electrons in a Möbius ring, is mentioned in the outlook but not developed. A qualitative contrast with the magnon case would enrich the discussion.

The manuscript is clearly written, physically insightful, and the theoretical results are carefully derived and clearly illustrated.

I **recommend acceptance with minor revisions**. The manuscript meets all criteria for *SciPost Physics*, including scientific novelty, clarity, and relevance. The results are solid, the idea is original, and the implications merit publication.

Minor comment: In several places, “eigenmodes of excitation” could simply be “eigenmodes” or “excitations” for conciseness.