In this paper, the authors show how products of Szegö kernels (which naturally arise, for instance, in correlators of fermions on Riemann surfaces) can be written in terms of either Enriquez kernels or the so-called DHS kernels, introduced by the authors in collaboration with Hidding in a previous work. A key advantage of these decompositions is that they separate the dependence on the spin structure from the dependence on the location of the fermions on the Riemann surface. This will certainly be useful in applications to string theory amplitudes in the RNS formalism.

The work is technical, but very clear: the relevant material is introduced systematically and examples are worked out explicitly, which will help readers with a physics background. More mathematically oriented readers will also benefit from this work, as they will find explicit proofs of the decompositions, including the one in terms of DHS-kernels, which was already proposed in a previous paper, but without a proof. It is clear from past work and work in progress mentioned i Sect. 6.4 that this paper is part of an interesting programme that the authors are pursuing. The results will be very useful for researchers interested in the study of string amplitudes and, more broadly, higher-genus polylogarithms. Thus I am happy to recommend this work for publication on SciPost.

In the attachment I include also some minor remarks that could be useful to the authors in case they plan to prepare a revised version of their manuscript.

- It is not immediately clear how to interpret the second line of Eq. (2.3) in the case $k = 1$: I guess that the delta (which for $k = 1$ would read $\delta_L^{I_r I_{r-1}}$) should be ignored.

- In order to reproduce some of the steps in Sect. 2, it would be useful to have the monodromies of the prime form: an equation recalling them could be added in Appendix A.1.

- I was slightly confused by the comment immediatly after Eq. (2.18): I would first see that result as obtaind directly from (2.17) and then I'd state that the combination $\partial_3 \chi^I(2,3) - \varpi^I{}_J(2) D^J_\delta(3)$ has the same monodromies (thanks to the equations quoted in the text). This implies that the combination mentioned three lines below Eq. (2.18) is single valued.

- It may be useful to recall after Eq. (2.38) the definition of the shuffle product (or just to mention that it appears in that equation).

- Sect. 7 is devoted to the study of linear chains of Szegö kernels. Is this sufficient to cover all structures that can appear in higher genus string amplitudes (when focusing on the even spin structures sector)? It may be useful to add a comment on this point.