EVALUATION ON "BCOV THEORY ON THE LARGE HILBERT SPACE"

String field theory has been a resource of rich algebraic and geometric structures in both physics and mathematics. The current article under evaluation provides a study of BCOV theory of Kodaira-Spencer gravity, which describes closed string fields in the B-twisted topological sector. Such theory connects complex geometry deeply with gravitational excitations on Calabi-Yau manifolds, and plays a fundamental role for the development of topological holography in recent years. The article presents a detailed study of BCOV theory from the viewpoint of spinning worldline with $N = (2, 2)$ supersymmetry on the source space, from which they obtain various superfield constructions of classical BCOV theory in the BV formalism. This result is useful to clarify how superstring field theory applies to topological sectors, and to potentially help understand the physics mechanism of topological holography. The paper is well-written with a few questions/comments that I will remark below. I **recommend** for publication in "SciPost Physics" after the authors reviewed them.

(1) In various places in the article, the author use "Tian's Lemma". It is commonly called "Bogomolov-Tian-Todorov" in the literature to credit main contributors.

(2) Page 4. The authors mentioned Costello-Li's formalism of BCOV theory as on-shell formalism. This is a misunderstanding. One key point of Costello-Li's formalism is the off-shell extension of Kodaira-Spencer gravity in terms of gravitational descendants. At the classical level, the off-shell Maurer-Cartan equation is equivalent to the classical BV master equation by an L_∞-transformation via Givental's Lagrangian cone construction.

(3) Page 10. For the pairing between positive and negative orders of γ, is there an additional sign which might play some role? As a comparison, in the Costello-Li's formulation, they use the symplectic space $PV(x)((u))$ where the symplectic pairing in the u-parameter is

$$\oint du f(u)g(-u)$$

In other words, it is a sesquilinear pairing where u^k is paired with $(-u)^{-k-1}$. Such sesquilinear pairing was introduced by K.Saito in his theory of higher residue, and plays an important role in Hodge theory. For detailed discussion on this, see the following review

- S.Li, *Some classical/quantum aspects of Calabi-Yau moduli*, B-Model Gromov-Witten Theory. Cham: Springer International Publishing, 2019. 463-497.

(4) The current work is mainly on classical aspect. As a side comment to communicate with the authors, I'm curious whether this method can be applied to solve some quantum case. As an example, in

- S.Li, *Vertex algebras and quantum master equation.* J.Diff.Geom. 123 (2023), no 3, 461 - 521.

a completion solution of quantum BCOV theory on elliptic curve is found which is related to quantum integrability. It would be extremely interesting to see whether this could be understood from the current method.