Probing New Physics Beyond ΛCDM with Large-Scale Structure

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Abstract

These lecture notes expand a 90-minute lecture delivered at the Les Houches Summer School "The Dark Universe" (July 2025). The goal is to give an overview of how we probe the imprints of new physics beyond ΛCDM with observations of the large-scale structure (LSS). Two case studies are discussed: (i) primordial non-Gaussianity (PNG) as a window into the field content and interactions during inflation; (ii) properties of neutrinos and light relics, namely the mass of neutrinos and the effective number of relativistic species. We outline why LSS is powerful for these questions, what we can learn, current bounds, the main analysis challenges, and prospects with ongoing and upcoming galaxy surveys. We will also illustrate the role of higher-order statistics in extracting non-Gaussian information of LSS and challenges on observational and theoretical fronts in obtaining robust and high-precision constraints on each of these imprints.

Provide the full form of Lambda CDM once

Contents

1 Introduction

Over the past three decades, cosmology has transformed into a truly *precision* science: full-sky measurements of the cosmic microwave background (CMB), wide-area galaxy surveys, and independent geometric probes—principally Type Ia supernovae and strong-lensing time delays—have reduced statistical uncertainties on the six parameters of the concordance ΛCDM model to the percent level or better. This quantitative success, however, sharpens rather than silences the field's most fundamental questions. The physical nature of dark matter (DM) and dark energy (DE) remains unknown; the mechanism that generated the nearly scale-invariant primordial fluctuations is yet to be fully understood; and the absolute mass scale of the neutrinos, together with the possible existence of additional light relics, continues to elude laboratory measurement~~s~~. Moreover, as uncertainties in several datasets shrink, tensions among them have emerged—for example in the values of the Hubble constant and the amplitude of matter clustering—hinting either at subtle systematic effects or imprints of new physics [1].

The three-dimensional distribution of matter and its tracers offers an especially powerful avenue for precision tests of fundamental physics. Having formed gravitationally from primordial fluctuations seeded during inflation, the LSS is sensitive to numerous possible extensions of ΛCDM, including primordial non-Gaussianity, total mass of neutrinos, and potential interactions among them and with other dark sector components, the effective number of relativistic species, the nature of DM and DE, and potential modifications to General Relativity on cosmological scales. Galaxy surveys map the LSS by measuring distribution and shapes of galaxies and quasars (clustering and weak lensing) in three dimensions, unlocking *orders-of-magnitude* more linear and quasi-linear Fourier modes than a purely two-dimensional CMB map. Adding spectroscopic redshifts—or, in photometric surveys, tomographic binning—encodes the time evolution of growth, opening sensitivity to dynamical effects as well as geometry.

Building on earlier successes, the current *fourth* generation of surveys is poised to test faint imprints of physics beyond the concordance model. DESI [2] already leads the spectroscopic effort, providing high-precision measurements of the expansion history and the growth of structure and, when combined with complementary data sets, delivering the tightest constraints to date on the ΛCDM model and several of its extensions [3]. ESA's Euclid mission [4] will complement these results with wide-field space-based imaging and slitless spectroscopy covering more than third of the sky; and NASA's SPHEREx mission [5] will add all-sky, low-resolution near-infrared spectra that supply both photometric redshifts and a spectral view of the diffuse infrared background. Deep ground-based imaging from the Rubin Observatory's LSST [6] will further enhance weak-lensing and clustering measurements across nearly half the sky. Joint studies that weave these datasets together with CMB measurements both suppress cosmic variance and cross-calibrate the distinct systematics of each probe, tying late-time structure to the physics encoded in the primary anisotropies. Looking ahead, line-intensity-mapping experiments [7, 8] represent the next frontier, measuring the statistical properties of large-scale structure across vast cosmic volumes and pushing studies to higher redshifts and fainter objects where the concordance model and its extensions remain virtually untested [9, 10].

In this data-rich landscape, mastering the theoretical framework, statistical methods, and observational systematics is essential, and the past few years have brought rapid progress on all three. Perturbative modeling of summary statistics (e.g., EFTofLSS [11–14]) remains the backbone of current pipelines, and fast, numerically stable evaluations now push reliable fits to smaller scales (e.g., [15]). Simulation-calibrated emulators (e.g., [16]) and simulation-based inference [17]—already applied to BOSS [18] and lensing data [19] but still more forward-looking—offer routes beyond the perturbative regime to harvest non-Gaussian information. Mitigation of observational systematics, e.g., imaging artifacts, fiber collisions, redshift failures, together with accurate non-Gaussian covariance modeling continues to set the practical precision ceiling; cross-correlations help diagnose residuals and reduce sample variance.

These lecture notes offer a broad orientation, prioritizing physical intuition and analysis strategy over detailed derivations, with an emphasis on spectroscopic galaxy surveys. In Section 2 we review the core workflow of LSS analyses–building the data vector from redshift surveys and forward-modeling the observable to infer cosmological constraints. Section 3 examines primordial non-Gaussianity, showing how deviations from Gaussian initial conditions imprint distinguishable signatures on late-time clustering and summarizing the methods used to extract them. Section 4 addresses neutrinos and light relics, detailing how free-streaming alters the growth and shape of structure and how upcoming surveys can probe the minimal mass scale. Finally, Section 5 offers an outlook on the remaining theoretical and observational challenges and on the prospects for leveraging next-generation data. Due to limited page count, I do not present any figures and refer the reader to the slides of this lecture publicly available on the website of Les Houches school.

2 Mapping the LSS in 3D with Spectroscopic Galaxy Clustering

In practice, cosmological information contained in galaxy catalogs is typically compressed into summary statistics, most commonly the N-point correlation functions (CFs). The nonlinear nature of structure formation means that the two-point CF alone provides a lossy compression of the data, and including statistics sensitive to the intrinsic non-Gaussianity of the galaxy field is essential. Although the full galaxy overdensity field contains additional information beyond 2- and 3-point CFs, field-level analyses and their application to large-volume surveys remain challenging and under active development. In this section, I briefly summarize the two most widely used galaxy clustering statistics—the power spectrum and bispectrum—within the ΛCDM framework, deferring discussions of imprints from PNG and neutrino properties to sections 3 and 4. We will not discuss here the estimators used to measure these statistics and the likelihood inference framework due to limitation in time.

The primary statistic used in galaxy clustering analyses has been the two-point CF, or equivalently its Fourier-space counterpart, the power spectrum, which at sufficiently large scales can be well-described by linear perturbation theory. On large-scales galaxies are biased tracers of the underlying dark matter distribution and the relation between the two can be describing via a constant bias coefficient $\delta_g(\mathbf{x},z) = b_1(z)\delta_m(\mathbf{x},z)$. Beyond linear limit, and on smaller scales the biasing relation becomes considerably more complex and nonlinear [20]. Another ingredient necessary in modeling galaxy clustering statistics is to account for the effect of peculiar velocities of galaxies (along their line of sight direction) on their clustering properties, i.e., the redshift-space distortions. Assuming Gaussian initial conditions and including redshift-space distortions at linear level, the anisotropic galaxy power spectrum is given by

Define vectors x, k and n

$$P_g^s(\mathbf{k},z) = [Z_1(\mathbf{k},z)]^2\, P_{\rm L}(k,z), \qquad (1)$$

s superscript?

where $P_{\rm L}(k,z)$ is the linear matter power spectrum, and $Z_1(\mathbf{k},z) = [b_1(z) + f(z)\mu^2]$ with $f(z)$ being the logarithmic growth rate and $\mu \equiv \hat{\mathbf{k}}\cdot\hat{\mathbf{n}}$ defining the line-of-sight direction. Deviations from ΛCDM modify the linear matter power spectrum, the growth rate, and—in certain cases—the biasing relation itself (e.g., local PNG; see section 3). Additionally, the observed galaxy power spectrum includes stochastic contributions, at leading order given by Poisson shot noise $P_{\rm shot}(z) = 1/\bar{n}(z)$, where $\bar{n}(z)$ is the galaxy number density. Moreover, comparisons between theory and observations must correct for geometric anisotropies (the Alcock–Paczyński effect) caused by differences between the assumed and true cosmology when converting teh observed angles and redshifts of galaxies into their 3-dimensional positions. In addition to modeling the clustering signal itself, careful treatment of survey-specific observational effects—including survey geometry [21, 22] and contamination by non-target galaxy populations [23]–is essential for obtaining robust cosmological constraints. In analyses, it is typically more convenient to use the Legendre multipoles of the anisotropic power spectrum:

$$P_\ell(k,z) = \frac{2\ell+1}{2}\int_{-1}^{1} d\mu\, P_g^s(k,\mu,z)\mathcal{L}_\ell(\mu), \quad \ell = 0,2,4,\dots\,, \qquad (2)$$

Defining L_l might be useful

with analyses usually restricted to $\ell \leq 4$ due to increasing statistical noise at higher orders.

Accurate extraction of clustering information into the weakly nonlinear regime demands sophisticated modeling beyond the linear expression above. The EFTofLSS framework (see e.g., [24] for a recent review) systematically incorporates nonlinear dark matter evolution, galaxy bias expansion, redshift-space distortions, infrared resummation of large-scale bulk flows—crucial for accurate modeling of baryon acoustic oscillations (BAO)—and stochastic contributions beyond simple Poisson noise. Additionally, EFTofLSS also includes counterterms that capture non-perturbative small-scale physics affecting large-scale power spectrum predictions. The resulting nonlinear model involves numerous nuisance parameters (biases, EFT coefficients, stochastic amplitudes) that must be marginalized over in cosmological analyses. When extending beyond ΛCDM, validating the assumptions underpinning these nonlinear modeling steps becomes even more critical.

Beyond power spectrum, higher-order statistics lift the degeneracies (among cosmological and nuisance parameters) that limit full-shape two-point analyses, in addition to lowering the cosmic variance, and giving access to the information carried by non-Gaussian mode couplings not captured by two-point statistics (see section 3). At tree-level, and again assuming Gaussian ICs, define IC the redshift-space galaxy bispectrum reads

$$B_g^s(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, z) = 2Z_1(\mathbf{k}_1, z)Z_1(\mathbf{k}_2, z)Z_2(\mathbf{k}_1, \mathbf{k}_2, z)P_L(k_1, z)P_L(k_2, z) + 2 \text{ cyc.}, \tag{3}$$

where Z_2 incorporates second-order density/velocity kernels and bias terms (see [25] for explicit expressions), and the wavevectors form closed triangles, i.e., $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}3 = 0$. Stochastic contributions to the bispectrum at leading order include Poisson noise terms:

$$B_{\text{shot}}(k_1, k_2, k_3, z) = \frac{P(k_1, z) + 2\text{cyc.}}{\bar{n}(z)} + \frac{1}{\bar{n}^2(z)}. \tag{4}$$

Similar to the case of power spectrum, to simplify the analysis, instead of analyzing the 5-dimensional bispectrum, commonly the following Legendre multipoles are analyzed,

$$B_\ell(k_1, k_2, k_3) = \frac{2\ell + 1}{4\pi} \int_0^{2\pi} d\phi \int_{-1}^{1} d\mu\, B_g^s(k_1, k_2, k_3, \mu, \phi)\mathcal{L}_\ell(\mu). \tag{5}$$

where we have chosen

$$\mathbf{k}_1 = (0, 0, k_1), \qquad \mathbf{k}_2 = (0, k_2 \sin\theta_{12} \cos\theta_{12}), \qquad \mathbf{n} = (\sin\theta_1 \cos\phi, \sin\theta_1 \sin\phi, \cos\theta_1). \tag{6}$$

The tree-level power spectrum and bispectrum models becomes inaccurate going beyond linear regime, i.e., with increasing k_{max}. Incorporating nonlinearities through one-loop corrections extends the analysis to smaller scales, enabling more precise cosmological constraints. Some recent analyses of BOSS and DESI data have included one-loop corrections to power spectrum to constrain the ΛCDM model and its minimal extensions. Considering the bispectrum, while some of the analysis of BOSS data included the one-loop corrections, a comprehensive treatment is very recent [26]. As an intermediate solution, minimally extending the tree-level redshift-space bispectrum by including EFT counterterms has been demonstrated to moderately extend the viable k_{max} Full stop

Despite their potential for significantly improving cosmological constraints, analyses using higher-order statistics remain challenging. These difficulties arise from theoretical modeling complexities, computational burdens stemming from the high dimensionality of observables (both in evaluating the signal and estimating the covariance), and practical challenges in treating observational systematics. Extending analyses beyond the bispectrum quickly becomes computationally prohibitive. Consequently, most clustering analyses to date have focused primarily on two-point correlation functions, with only a few studies incorporating higher-order statistics. This limitation has motivated efforts to compress the information contained in higher-order N-point functions into simpler, lower-dimensional observables. Examples of such compressed statistics include line and phase correlation functions, position-dependent power spectrum, marked power spectra, weighted skew and kurto spectra [27–33]. Additionally, alternative summary statistics designed explicitly to extract non-Gaussian information from LSS have been considered, including wavelet scattering transforms [34] and density-split clustering [35]. Some of these statistics are particularly interesting since they can be tailored to enhance sensitivity to particular signatures of new physics; for instance, skew spectra for primordial non-Gaussianity [36], kurto spectra for parity-odd trispectra [37], and marked power spectra for neutrino mass constraints [38].

3 Primordial Non-Gaussianity in LSS

Having discussed the very basics of galaxy clustering analysis, this section develops how inflation leaves testable imprints in late-time clustering, with an emphasis on primordial non-Gaussianity (PNG). We recall why PNG is a decisive target for beyond-minimal inflation, connect theory to the observables used in spectroscopic surveys, summarize current bounds and prospects, and close with analysis challenges and mitigation strategies.

3.1 Theoretical background

Inflation was originally proposed to solve the horizon and flatness problems of standard Big Bang cosmology: a period of accelerated expansion drives the spatial curvature toward zero and stretches any pre-existing inhomogeneities outside the causal horizon. However, the most powerful and testable prediction of inflation lies in the quantum generation of primordial fluctuations, including tensorial and scalar ones. Vacuum fluctuations of the inflaton (or additional light fields) are stretched to cosmological scales, seeding curvature perturbations $\zeta(\mathbf{x})$ whose statistics reflect the microphysics at energies near the inflationary Hubble scale H and set the initial conditions for the observed CMB anisotropies and all the large-scale structure. inflation

In the simplest models of inflation—assuming Bunch-Davies vacuum and ~~driven~~ by a single scalar degree of freedom (e.g., described by a scalar field with a flat potential and canonical kinetic term)—the scalar fluctuations are predicted to be nearly Gaussian and scale-invariant, well-characterized by a power-spectrum P_ζ described by an amplitude A_s and a tilt n_s,

$$\Delta^2_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1} .$$

Here, P_ζ is defined as $\langle \zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle = (2\pi)^3 \delta_D(\mathbf{k}+\mathbf{k}') P_\zeta(k)$ and $k_\star$ is a chosen pivot scale. Observations of the CMB and LSS to date confirm this prediction with remarkable precision. However, as survey volumes and measurement accuracy continue to grow, even subtle departures from scale-invariance and Gaussianity of primordial fluctuations can be detected by searching for “primordial features" and “primordial non-Gaussianities" [39]. Three-dimensional clustering of galaxies, which is our focus here, provides an exceptionally powerful laboratory for these searches, complementing primary and secondary CMB anisotropies.

The rest of our discussion will be focused on PNG, which encodes microphysics of inflation: field content, interactions, and initial state. Different inflation models differ in their predictions for 3- and 4-point correlation functions of ζ (bispectrum and trispectrum). For translation-, rotation- and scale-invariant perturbations, the bispectrum for a given type of primordial non-

Gaussianity is often described in terms of an amplitude $f_{\rm NL}^{(\rm type)}$ and a shape function $S^{(\rm type)}$,

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,\frac{18}{5}\,f_{\rm NL}^{(\rm type)}A_s^2\,\frac{S^{(\rm type)}(k_1,k_2,k_3)}{(k_1k_2k_3)^2}. \tag{7}$$

Broad classes of inflationary models give rise to specific shapes for the bispectrum. Three most-commonly-studied ones are local, equilateral, and orthogonal shapes. A robust detection of $|f_{\rm NL}|\gtrsim 1$ in any of these shapes would thus reveal new fields or interactions at play during inflation, opening a unique window onto physics at the highest energy scales.

For example, a large *local*-type bispectrum indicates the presence of more than one light field (with mass $m \ll H$) during inflation since according to a consistency relation between the 3 and 2 point functions of ζ in single-field models [40],

$$\lim_{k_L\ll k_S} B_\zeta(k_L,k_S,k_S) = (1-n_s)P_\zeta(k_L)P_\zeta(k_S) + \mathcal{O}[(k_L/k_S)^2], \tag{8}$$

which implies $f_{\rm NL}^{\rm loc}\sim\mathcal{O}(10^{-2})$ given current bounds on n_s. A detection of $|f_{\rm NL}^{\rm loc}|\gtrsim 1$ will therefore rule out single-field models of inflation and is therefore a smoking gun for additional light particles during inflation. As we report below, Planck data has constrained $f_{\rm NL}^{\rm loc}$ to be order unity, and the expectation is that the current generation of LSS data can improve upon this constraint considerably.

Single-field inflation models, however, can generate other types of PNG. For instance if inflaton has derivative interactions, several bispectrum shapes can be produced depending on the form of the derivatives. Since derivatives are suppressed on superhorizon scales, these interactions contribute the most to the PNG signal when all modes have similar wavenumbers around the horizon exit. Therefore these shapes peak in the equilateral configuration ($k_1\sim k_2\sim k_3$). Within the EFT of single-field inflation [41] and considering cubic interactions, there are two operators, which give rise to two different bispectrum shapes, that are commonly represented by two nearly orthogonal templates (*equilateral* and *orthogonal* shapes).

While the presence of light scalar fields gives rise to local-type bispectrum, if particles of mass $m\sim H$ and spin s are present during inflation, their exchange imprints characteristic non-analytic momentum and angular dependence in the squeezed-limit bispectrum that is well-approximated by the form [42]

$$\lim_{k_L\ll k_S} B_\zeta(k_L,k_S,k_S) \propto \left(\frac{k_L}{k_S}\right)^{3/2+\nu}\mathcal{L}_s(\hat{\mathbf{k}}_L\cdot\hat{\mathbf{k}}_S)P_\zeta(k_L)P_\zeta(k_s) \tag{9}$$

$$\text{where}\qquad \nu\equiv\sqrt{\left(s-\frac{1}{2}-\delta_{s0}\right)^2-\frac{m^2}{H^2}}$$

long

where k_L and k_S represent the lang- and short-scale fluctuations, while $\mathcal{L}_s$ are the Legendre polynomials which encode the spin-s angular dependence. This class of PNG is broadly referred to as *cosmological collider types*, and if detected, can tell us about the mass and spin of particles during inflation. For $s=0$ for example, if $m>3H/2$ (imaginary ν), the squeezed limit bispectrum acquires logarithmically oscillatory behavior. Detecting this signal would thus measure the mass and spin of particles at inflationary energies.

Beyond the bispectrum, higher–point functions reveal additional facets of inflationary physics. Contact trispectra arise from quartic self-interactions and are parametrized by an amplitude $g_{\rm NL}$, while exchange trispectra—generated by particle exchange during inflation—are characterized by the collapsed-limit amplitude $\tau_{\rm NL}$. In multi-field local models these satisfy the Suyama–Yamaguchi inequality $\tau_{\rm NL}\geq(6/5f_{\rm NL}^{\rm loc})^2$ [43], implying that a large local bispectrum necessarily produces an enhanced trispectrum. However, there exist scenarios (for instance those with symmetry-protected cubic interactions) in which the bispectrum is suppressed while the trispectrum remains sizable. An especially intriguing class of models breaks parity during inflation—since a scalar bispectrum is insensitive to parity [44], the trispectrum becomes the lowest-order statistic that can carry a parity-odd signature. Beyond correlators, the extreme tails of the ζ probability distribution can also probe nonperturbative inflationary dynamics: rare, large fluctuations—sourced by resonant particle production or barrier-crossing events—produce heavy, non-Gaussian tails with distinctive signatures on LSS observables [45].

3.2 Observational imprints on LSS

Non-Gaussian correlations in the primordial curvature field are directly inherited by the late-time matter field, since $\zeta(\mathbf{k})$ sets the initial conditions for $\delta_m(\mathbf{k})$:

$$\delta_m^{\rm lin}(\mathbf{k},z) = \mathcal{M}(k,z)\zeta(\mathbf{k}), \qquad \mathcal{M}(k,z) = \frac{2}{5}\frac{k^2T(k)D(z)}{\Omega_m H_0^2} \tag{10}$$

where $T(k)$ is normalized to unity on large scales and $D(z)$ is the linear growth factor. Hence, at linear order the primordial bispectrum $B_\zeta^{\rm type}$ maps into a matter bispectrum

$$B_m^{\rm PNG}(k_1,k_2,k_3;z) = \mathcal{M}(k_1,z)\mathcal{M}(k_2,z)\mathcal{M}(k_3,z)B_\zeta^{\rm type}(k_1,k_2,k_3). \tag{11}$$

Even for Gaussian initial conditions, nonlinear gravitational evolution produces a matter bispectrum that—under current bounds—typically exceeds $B_m^{\rm PNG}$, motivating our use of the superscript “PNG” to isolate the primordial component. The large gravitationally-induced late-time bispectrum makes constraining PNG from LSS more challenging than from the CMB which is well described by linear theory and less prominent late-time bispectrum coming from secondary CMB anisotropies.

In addition to producing non-zero matter N-point functions, some types of PNG can also modulate the abundance and clustering of biased tracers such as halos and galaxies. In particular, local-type PNG induces a coupling between long- and short-wavelength modes: large-scale potential fluctuations ϕ_l modulate the small-scale variance and hence the halo/galaxy abundance. Within perturbative description of biased tracers, the effect on clustering can be described by extending the bias expansion to include operators constructed from primordial gravitational potential at early times. At leading order, the bias expansion is given by

$$\delta_g(\mathbf{x},z) = b_1(z)\delta_m(\mathbf{x},z) + f_{\rm NL}^{\rm loc} b_\phi(z)\phi(\mathbf{q}), \tag{12}$$

where b_ϕ is the PNG-bias coefficient, $\mathbf{x}$ is the late-time (Eulerian) position of galaxies/DM vs $\mathbf{q}$ is the initial (Lagrangian) position, which are related to one another by displacement field of DM. Using $\phi(\mathbf{k}) \propto \mathcal{M}^{-1}(k)\delta_m(\mathbf{k})$ gives the well-known scale-dependent bias [46]

$$\Delta b_1(k,z) = f_{\rm NL}^{\rm loc} b_\phi(z)\mathcal{M}^{-1}(k,z) \propto k^{-2}, \tag{13}$$

which enhances or suppresses the galaxy power spectrum on the largest scales depending on the sign of $f_{\rm NL}^{\rm loc}$. Beyond the linear limit, the non-Gaussianity present in the initial conditions couples to gravitational evolution, and modifies the growth of matter perturbations as well. Therefore, going beyond linear order requires consistently including higher-order bias operators constructed from matter density and tidal field, the primordial gravitational potential and cross-terms among them [20, 47].

In the present of local PNG, the tree-level galaxy power spectrum in redshift-space is thus given by

$$P_g^s(\mathbf{k},z) = \left[Z_1^{\rm tot}(\mathbf{k},z)\right]^2 P_m(k,z), \tag{14}$$

where

$$Z_1^{\rm tot}(\mathbf{k},z) = Z_1(\mathbf{k},z) + Z_1^{\rm PNG}(\mathbf{k},z) = b_1(z) + f(z)\mu^2 + f_{\rm NL}^{\rm loc} b_\phi(z). \tag{15}$$

The galaxy bispectrum acquires two distinct PNG contributions at tree level: (i) the mapped primordial matter bispectrum $B_m^{\rm PNG}$, and (ii) additional mode-couplings from PNG bias operators. So the tree-level galaxy bispectrum is now given by

$$B_g^s(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,z) = \left[Z_1^{\rm tot}(\mathbf{k},z)\right]^3 B_m(k_1,k_2,k_3,z) \\ + 2Z_1^{\rm tot}(\mathbf{k}_1,z)Z_1^{\rm tot}(\mathbf{k}_2,z)Z_2^{\rm tot}(\mathbf{k}_1,\mathbf{k}_2,z)P_L(k_1,z)P_L(k_2,z) + 2\text{ cyc.}, \tag{16}$$

where the $Z_2^{\rm tot} = Z_2 + Z_2^{\rm PNG}$ with Z_2 being the standard 2nd-order mode coupling kernel including 2nd-order kernels of matter density and velocity, and galaxy biases) and $Z_2^{\rm PNG}$ contains new contributions due to PNG bias operators at 2nd order (see e.g., [48] for the explicit expression of $Z_2^{\rm tot}$). Note that since in the expression of Z_2 in [48], only terms linear in $f_{\rm NL}^{\rm loc}$ are retained motivated by the fact that current data constrains local-type bispectrum to be small. To be consistent with this logic, in Eq. (16), nonlinear terms in $f_{\rm NL}^{\rm loc}$ should also be discarded.

For other types of primordial bispectrum, the form of the additional operator in eq. (12) is modified and can be determined from the squeezed-limit of the primordial bispectrum (see section 7.1.3 of [20] for further details). For equilateral and orthogonal shapes, the correction to large-scale bias is scale-independent and thus degenerate with the Gaussian linear bias b_1. While on smaller scales, these types leave an imprint on galaxy power spectrum (via scale-dependence of transfer function and one-loop corrections from matter bispectrum), the effects are nearly indistinguishable from scale-dependencies due to various nonlinear effects present for Gaussian initial conditions. Therefore, constraining PNG of equilateral and orthogonal types, i.e., single-field inflation models relies on measurement of galaxy bispectrum, which now only receives the PNG contribution only through the first term in Eq. (16).

As we will see below, current LSS bounds on local PNG are dominated by the large-scale galaxy and quasar power spectra. Nevertheless, addition of the galaxy bispectrum has been shown to improve the power spectrum constraint by nearly 20%. The recent LSS constraints on other shapes [49, 50] has relied on the galaxy bispectrum.

As we will discuss in section 3.3, a key practical hurdle in measuring local PNG from the galaxy power spectrum is the near-perfect degeneracy between the amplitude and the PNG bias coefficient in Eq.(12). Since both enter observables only through the product , one cannot separately constrain without fixing or imposing an external prior. In principle, one-loop corrections to the galaxy power spectrum arising from the primordial matter bispectrum contribute a term independent of PNG bias, but these corrections are highly suppressed and thus fail to break the degeneracy. At the bispectrum level, linear and quadratic PNG bias exhibit analogous degeneracies with $f_{\rm NL}^{\rm loc}$. In this case, the contribution of matter bispectrum (first line of Eq. (16)), which is independent of PNG biases mitigates the degeneracies. Nevertheless, robust inference of $f_{\rm NL}^{\rm loc}$ still requires externally imposed priors on the PNG biases—whether derived from tailored simulations of the galaxy sample or calibrated directly from data (or cross-correlations with other probes).

3.3 Current bounds and future prospects

Currently, the tightest bounds on PNG come from Planck's CMB bispectrum, which constrains the local, equilateral, and orthogonal amplitudes at the few-tens level. Large-scale structure surveys are rapidly catching up: DESI's first data release has already more than halved the LSS uncertainties $f_{\rm NL}^{\rm loc}$, and other Stage-IV surveys are expected to reduce errors by factors of several. Here are direct comparison of representative current limits:

$$\textbf{CMB - Planck [51]:}\quad f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1, \quad f_{\rm NL}^{\rm eq} = -26 \pm 47, \quad f_{\rm NL}^{\rm orth} = -38 \pm 24. \tag{17}$$

BOSS DR12 [48, 49]:
$(0.2 < z < 0.8)$

$$P + B : f_{\rm NL}^{\rm loc} = -33 \pm 28,$$
$$B : f_{\rm NL}^{\rm eq} = 260 \pm 300, \quad B : f_{\rm NL}^{\rm orth} = -23 \pm 120. \tag{18}$$

eBOSS DR16 QSO [52, 53]:
$(0.8 < z < 2.2)$

$$\text{P:} \quad -4 < f_{\rm NL}^{\rm loc} < 27,$$
$$\text{P+B:} \quad -6 < f_{\rm NL}^{\rm loc} < 20. \tag{19}$$

DESI DR1 (LRG+QSO) [54]:
(LRG $0.4 < z < 1.1$; QSO $0.8 < z < 2.1$)

$$P : f_{\rm NL}^{\rm loc} = -3.6^{+9.0}_{-9.1};$$
$$\text{LRG:} \ f_{\rm NL}^{\rm loc} = 6^{+22}_{-18}, \quad \text{QSO:} \ f_{\rm NL}^{\rm loc} = -2^{+11}_{-10}. \tag{20}$$

Ongoing Stage-IV spectroscopic surveys (DESI, Euclid) are poised to transform searches for PNG through a combination of survey volume, redshift reach, and overlapping tracers. They are on track to match Planck's CMB-only bounds with power-spectrum measurements and—by leveraging the bispectrum—could push the sensitivity to $f_{\rm NL}^{\rm loc}$ down to $\mathcal{O}(1)$. Meanwhile, the *SPHEREx* mission—with full-sky near-IR spectroscopic mapping and multi-tracer splitting of galaxy samples that enables cosmic-variance cancellation on the ultra-large scales most relevant for local PNG [55]—promises to tighten both power-spectrum and bispectrum constraints further. Constraints on other PNG shapes beyond the local type (e.g., equilateral, orthogonal, cosmological-collider), which rely on bispectrum analyses, are likewise expected to improve substantially with Stage-IV data, contingent on theoretical modeling that matches the precision of data—namely, accurate treatment of gravitational nonlinearities (e.g., EFTofLSS counterterms and IR resummation) and the PNG contributions themselves.

In addition, cross-correlations with CMB lensing from CMB surveys offer a promising avenue for obtaining robust constraints on PNG. CMB lensing probes the underlying dark matter distribution through its deflection of CMB photons, while galaxy and quasar clustering trace the same matter field but with different observational systematics. Their cross-correlation, particularly when combined with galaxy or quasar auto-spectra, provides a valuable tool in the systematics-limited regime: it helps calibrate large-scale bias and imaging residuals, reduces sample variance, and enables access to the largest scales with controlled window functions. A recent analysis using the DESI LRG spectroscopic sample and Planck lensing maps [56] reported $f_{\rm NL}^{\rm loc} = 39^{+40}_{-38}$ from the cross-correlation alone, and $f_{\rm NL}^{\rm loc} = 24^{+20}_{-21}$ when combined with the LRG auto-spectrum. A similar analysis using DESI spectroscopic quasars and Planck CMB lensing [57] found consistent results and emphasized the relative stability of the cross-correlation signal against imaging systematics. Both studies highlight the value of joint analyses in improving PNG constraints, and anticipate that future analyses using the full DESI spectroscopic dataset will lead to tighter bounds. In addition, upcoming Euclid data will offer further opportunities for cross-correlations between emission line galaxies and CMB lensing.

Looking beyond $z \simeq 2$, both high-redshift spectroscopic surveys and line-intensity mapping open vast new volumes for PNG searches. Proposed galaxy programs such as MegaMapper [58] $(2 < z < 5)$ can trace an order of magnitudes more spectroscopic targets than DESI for instance over tens of thousands of square degrees, directly accessing near-linear modes at early times. At the same time, wide-field LIM experiments (e.g. PUMA for 21cm [59], next-generation mm-LIM [10]) promise ultra-deep, low-noise maps of the same redshift range, potentially making them the ultimate probes of post-reionization non-Gaussianity. Bundling these two approaches—high-z galaxies and LIM—will not only shrink statistical errors on $f_{\rm NL}^{\rm loc}$ by exploiting complementary systematics and scale dependences, but also provide critical cross-checks to ensure robustness against foregrounds, tracer bias uncertainties, and instrumental effects.

3.4 Analysis challenges

Measurements of $f_{\rm NL}$ of different shapes from LSS face a variety of challenges across observational, theoretical, and data analysis fronts. Depending on the PNG type and its observational signature, some challenges are more pronounced than others.

For local PNG, which induces a coupling between long- and short-wavelength fluctuations, large-scale observational systematics represent a primary limitation—particularly for power spectrum constraints, but also for bispectrum analyses. Squeezed bispectrum configurations, which carry most of the constraining power for local PNG, are equally sensitive to these large-scale systematics. In addition to observational limitations, two theoretical challenges affect the interpretation of large-scale modes. First, as discussed earlier, a near-perfect degeneracy exists between $f_{\rm NL}^{\rm loc}$ and the PNG galaxy bias parameter b_ϕ, implying that $f_{\rm NL}^{\rm loc}$ cannot be constrained without a prior on b_ϕ. Including the bispectrum helps mitigate this degeneracy but does not eliminate the need for a well-motivated prior. Second, general-relativistic projection effects [60–62] can induce a scale-dependent enhancement on large scales that mimics the signature of local PNG, which must be accurately modeled to avoid false detections or biased constraints.

For equilateral and orthogonal shapes, constraints rely primarily on the bispectrum, which introduces additional analysis challenges. First is the difficulty of accurately modeling the bispectrum signal. Using the EFTofLSS perturbative framework allows systematic inclusion of non-linear effects, but it introduces a large number of nuisance parameters that are often degenerate with each other and with cosmological parameters. Extracting robust constraints requires astrophysical insight to place meaningful priors on these parameters. Beyond modeling of bispectrum signal, including observational effects such as survey geometry [63] and contamination from interloper galaxies [23] must be incorporated. Compared to the power spectrum, modeling these effects in the bispectrum is significantly more complex. While approximate treatments exist, their accuracy remains untested for the volume and precision of current surveys. In addition, bispectrum analyses are computationally demanding due to the high dimensionality of the data vector. This complicates both likelihood evaluations and, more critically, the estimation of covariance matrices. Accurate bispectrum covariance requires a large number of mock catalogs, far exceeding the needs of power spectrum analyses. As noted earlier, an active area of development is the design of lower-dimensional statistics that compress bispectrum information—such as skew spectra [64]—to make analyses more tractable.

For PNG shapes beyond the most commonly studied cases—such as those predicted by cosmological collider models—several further developments are required. First, constructing separable bispectrum templates that capture the key features of exact predictions across different model classes is essential for practical likelihood analyses [36, 65]. Second, the distinctive features of these bispectra—such as subtle oscillatory patterns that encode the mass and spin of particles during inflation—require careful modeling, particularly in the context of non-linear structure formation. Finally, the impact of observational systematics on these signatures must be carefully assessed, as their unique features may be especially susceptible to observational artifacts and modeling uncertainties.

4 Neutrinos and light relics in LSS

This section explores how neutrinos and other light relics affect LSS observables, from their impact on the expansion history to their suppression of small-scale structure growth. We briefly review the theoretical framework connecting neutrino properties to large-scale structure, summarize current constraints and future prospects, and close with a discussion of analysis challenges and possible systematics.

4.1 Theoretical background

Neutrinos are among the most abundant particles in the Universe, and thus affect different epochs in the cosmic history. Cosmological observations are sensitive to the effective number of neutrinos, $N_{\rm eff}$ when they were still relativistic and contributed to the radiation content of the Universe, as well as their total mass, $\sum m_\nu$ when they became non-relativistic and contribute to the matter content [66].

Neutrino oscillation experiments have conclusively established that at least two mass eigenstates are nonzero, via measurements of two independent mass–squared splittings. These allow two possible hierarchies—normal and inverted—setting the minimum sums $\sum m_\nu \gtrsim 0.058\,\mathrm{eV}$ (normal) and $\sum m_\nu \gtrsim 0.10\,\mathrm{eV}$ (inverted). Several key questions remain: the absolute mass scale, the ordering, and the mechanism of neutrino mass generation. Cosmology is sensitive to the absolute mass because massive neutrinos affect both the background expansion and the growth of structure through free–streaming. A cosmological detection or limit near the normal–hierarchy floor, combined with oscillation data, can determine the ordering and inform models of mass generation.

Even if a species is not in thermal contact with standard model it will contribute to radiation energy density and affect the expansion rate

Beyond the mass, $N_{\rm eff}$ probes additional light relics—sterile neutrinos, axion–like particles, or hidden–sector states once in thermal contact with the Standard Model. A deviation $\Delta N_{\rm eff} \neq 0$ would be direct evidence for new relativistic degrees of freedom. Complementarity is essential: the CMB constrains pre–recombination expansion and acoustic physics but suffers degeneracies with other cosmological parameters (notably amplitude of primordial fluctuations and optical depth of CMB photons; LSS adds low–redshift geometry and growth (BAO, RSD, full–shape), and cross–correlations with CMB lensing calibrate large–scale bias and reduce sample variance.

4.2 Observational imprints on LSS

Standard–Model neutrinos are relativistic in the early universe and become nonrelativistic when $T_\nu \simeq m_\nu$, around $z_{\rm nr} \simeq 1890\,(m_\nu/\mathrm{eV})$. Their present–day density is

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{93.14\ \mathrm{eV}}, \qquad f_\nu \equiv \frac{\Omega_\nu}{\Omega_m} \ll 1\,, \tag{21}$$

while in the relativistic regime they contribute to the radiation density and their energy density is determined by their effective number and temperature,

$$\rho_\nu(z) = \frac{7}{8}\frac{\pi^2}{15} N_{\rm eff}\, T_{\nu,0}^4\,(1+z)^4, \qquad T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0}, \tag{22}$$

where $T_{\nu,0}$ is their present-day temperature. When non-relativistic, their energy density is proportional to their total mass

$$\rho_\nu = \Omega_\nu \rho_{c,0}(1+z)^3 = \left(\frac{\sum m_\nu}{93.14\ \mathrm{eV}\ h^2}\right)\left(\frac{3H_0^2}{8\pi G}\right)(1+z)^3 \tag{23}$$

Thus, both relativistic and non-relativistic neutrinos affect the expansion history and are imprinted on geometric cosmological probe like BAO. Changes in $N_{\rm eff}$ additionally imprint a *BAO phase shift*: free–streaming radiation alters the driving of acoustic oscillations and shifts the peak positions in a way that is preserved by nonlinearity and bias, yielding a particularly clean LSS signature of $\Delta N_{\rm eff}$ [67]. The phase has been measured in existing data [68, 69]. Nonetheless, full–shape information remains valuable: broadband clustering encodes complementary $N_{\rm eff}$ dependence beyond the phase alone [70].

For $\sum m_\nu$, the BAO measurements constrain the total matter density and distances but cannot by themselves isolate the neutrino mass from CDM. Breaking this degeneracy requires combination with other probes—most effectively with CMB temperature and polarization—which leads to the tightest current bounds (see section 4.3). The main LSS signature of massive neutrinos is the suppression of growth on scales below the comoving free–streaming length. Writing the thermal velocity as $v_{\rm th}(z) \simeq 3.15 T_\nu(z)/m$ and the Hubble rate as $H(z)$, the free–streaming scale is

$$k_{\rm fs}(z) \equiv \sqrt{\frac{3}{2}\frac{H(z)}{v_{\rm th}(z)}}, \tag{24}$$

and modes with $k \gg k_{\rm fs}$ experience reduced clustering in the neutrino component, giving the linear suppression

$$\frac{\Delta P_m}{P_m} \simeq -8 f_\nu \qquad (k \gg k_{\rm fs}) \tag{25}$$

with a smooth transition near $k \sim k_{\rm fs}$. This scale–dependent suppression induces a characteristic redshift-dependent tilt in galaxy power–spectrum multipoles $P_\ell(k)$, but P_ℓ—only fits are limited by degeneracies with A_s/σ_8, n_s, Ω_m, and bias parameters. Redshift–space anisotropies space (through the μ–dependence)ameliorate the degeneracies. Galaxy bispectrum B_g provides additional constraining power because its mode couplings and redshift scaling respond differently to massive neutrinos [71, 72].

Modeling these effects requires a consistent treatment of the late–time matter components: baryons, CDM, and neutrinos. A full multi–fluid description [73, 74] is in principle necessary, but on the mildly nonlinear scales used in galaxy–clustering analyses, baryons and CDM remain tightly coupled and neutrino perturbations remain small. In this regime, a one–fluid EFTofLSS description of the CDM+baryon ("cb") field, with neutrinos treated linearly, captures the relevant nonlinear dynamics. The total matter contrast is then

$$\delta_m = (1 - f_\nu)\delta_{cb} + f_\nu \delta_\nu, \quad \delta_\nu(k,z) = \frac{T_\nu(k,z)}{T_{cb}(k,z)}, \delta_{cb}(k,z), \tag{26}$$

where T_ν and T_{cb} are the linear transfer functions. Because $\delta_{cb}(k,z)$ is no longer separable in k and z, exact time– and scale–dependent kernels are costly to compute; in practice, most EFTofLSS implementations use Einstein–de Sitter (EdS) kernels with the full scale dependence of the linear growth factor from a Boltzmann code, neglecting the much smaller neutrino–induced scale dependence in the growth rate. An improved approximation that retains the dominant corrections to EdS kernels was proposed in [75].

For biased tracers such as galaxies, the large neutrino velocity dispersion prevents them from clustering on halo–formation scales, making it more accurate to write the bias expansion in terms of δ_{cb} rather than δ_m [76]. Under this assumption and the above approximations, one–loop galaxy power–spectrum and bispectrum calculations can be extended to massive–neutrino cosmologies.

4.3 Current bounds and future prospects

Measurements of CMB anisotropies combined with those of BAO currently provide the tightest bounds on $\sum m_\nu$, with full–shape galaxy clustering delivering complementary constraints and consistency checks. More specifically, the most stringent limits to date come from DESI DR2 BAO combined with CMB datasets (including CMB lensing) [77]. In flat ΛCDM with three degenerate neutrinos and the physical prior $\sum m_\nu > 0$, the bound is $\sum m_\nu < 0.0642$eV (95% C.L.). Allowing a time–dependent dark–energy equation of state and considering $w_0 w_a \Lambda$CDM model, relaxes the constraint to $\sum m_\nu < 0.16$ eV (95% C.L.) given the well–known degeneracy between $\sum m_\nu$ and (w_0, w_a). These results are close to the neutrino oscillation experiment's lower bounds and would exclude the inverted hierarchy in ΛCDM if taken at face value.

However, this bound exhibits curious features: the marginalized posterior peaks at the prior boundary $\sum m_\nu = 0$, resembling the tail of a distribution with central value in the negative mass range; a profile–likelihood analysis indicates the parabola's minimum lies in the unphysical negative–mass region when extrapolate, underscoring the need for caution and for further cross–checks as data improve. Upcoming DESI releases should clarify whether the preference reflects new physics or residual systematics/tensions in the combined datasets.

For $N_{\rm eff}$, DESI DR2 BAO + CMB yields constraints consistent with the Standard Model. In the one–parameter extension ΛCDM+$N_{\rm eff}$, the combined data give $N_{\rm eff} = 3.16^{+0.35}_{-0.34}$ (95% C.L.). latex typo When varying Neff and $\sum m_\nu$ simultaneously, the correlation between them is weak, so the $N_{\rm eff}$ error degrades only marginally. DESI has also measured the BAO phase shift—the cleanest late–time imprint of free–streaming radiation—with a $\sim 2.2\sigma$ detection, consistent with the Standard Model expectation, and at higher significance compared to the BAO measurement from DESI-DR1 [69].

Looking ahead, DESI's full five–year BAO sample combined with next–generation CMB experiments (e.g., CMB–S4 [78]) is forecast to reach sensitivity to the normal–hierarchy mass scale at higher significance. Euclid's spectroscopy and weak–lensing imaging add complementary geometry and growth information; forecasts of [79] indicate Euclid+Planck can provide at least $\sim 2\sigma$ evidence for nonzero $\sum m_\nu$, rising to $\sim 4\sigma$ when combined with Stage–IV CMB data. Including higher–order statistics in robust full–shape pipelines should further tighten constraints relative to two–point analyses alone [72, 80]. In summary, within the next 5-10 years, the combination of DESI, Euclid, and next-generation CMB data should measure the neutrino mass; a null result would challenge the standard cosmological and particle-physics framework, hinting at new neutrino properties. In either case, the coming years promise to be an extremely exciting period for neutrino cosmology and fundamental physics.

For $N_{\rm eff}$, CMB measurements will continue to provide the most competitive individual constraints, but combining them with galaxy–clustering data yields notable gains, particularly when the cosmological parameter space is extended to include quantities highly degenerate with $N_{\rm eff}$—most prominently the primordial helium fraction, which significantly limits CMB–only determinations. Joint analyses of DESI and Euclid data with Stage–IV CMB surveys are projected to achieve an order–of–magnitude improvement over current precision, potentially detecting or excluding the minimal abundance of thermal extra light relics ($\Delta N_{\rm eff} \geq 0.027$) at high significance. High–redshift tracers such as line–intensity mapping can further enhance sensitivity by extending the redshift lever arm into the matter–radiation transition epoch, where the fractional impact of additional relativistic species on the expansion rate is largest [9, 81].

4.4 Analysis challenges

Pushing neutrino constraints into the sub-0.1 eV regime requires exquisite control of both statistical and systematic uncertainties. A key challenge is the statistical interpretation of bounds near the prior boundary, making the credible intervals highly sensitive to prior choice. Tight upper limits on total mass can reflect noise fluctuations, residual tensions between datasets, or subtle modeling biases rather than a genuine preference for very low mass.

On the modeling side, extracting robust information from large–scale structure requires an accurate nonlinear framework that incorporates the scale–dependent growth induced by free–streaming neutrinos while controlling theoretical systematics in galaxy–clustering and lensing observables. This includes properly modeling redshift–space distortions, non-linear bias, mode coupling, and relevant observational effects at the level demanded by next–generation precision. Theoretical uncertainties in these ingredients can bias $\sum m_\nu$ constraints if not consistently propagated through parameter inference. In EFT-based modeling approach, care is required to ensure that scale–cuts, loop corrections, and nuisance marginalization do not artificially tighten constraints, while in analysis based on simulation-calibrated emulators, various approximations in implementation of neutrinos should be scrutinized.

Another difficulty is parameter degeneracy in extended cosmological models. Allowing time–dependent dark energy significantly broadens the $\sum m_\nu$ posterior, as these parameters imprint similar signatures on background distances. Finally, cross–survey combinations must reconcile differences in systematics, calibration, and astrophysical modeling—challenges that will only grow as datasets increase in size and precision.

5 Conclusion and Outlook

Precision measurements of statistical properties of the large-scale structure by spectroscopic and photometric galaxy surveys offer a powerful mean s to probe fundamental physics well beyond the reach of laboratory experiments, from the particle content of the early Universe to the properties of the lightest massive particles known. The field has matured rapidly, with theoretical advances in EFTofLSS enabling robust modeling of non-linear scales and higher-order statistics, and the prospects of analyzing large volumes of high-precision data using simulation-calibrated emulators or full simulation-based inference is becoming more achievable. Concurrently, observational capabilities have expanded dramatically, from early surveys measuring thousands of redshifts to current Stage-IV experiments observing millions of galaxies.

redshifts or galaxies?

In these lectures we have reviewd two key frontiers where these developments have the greatest potential impact:

- **Primordial non–Gaussianity** encodes the detailed microphysics of inflation. Local–type PNG leaves a characteristic k^{-2} scale–dependent bias on large scales, while equilateral and orthogonal shapes require accessing the squeezed-to-equilateral mode couplings in higher-order statistics. Theoretical control of bias expansions, loop corrections, and survey systematics is essential for extracting the small, correlated signal across tracers and redshifts. Reaching $\sigma(f_{\rm NL}) \lesssim 1$ demands exploiting all available statistics—two- and three-point functions, cross-correlations, and multi–tracer strategies—while rigorously propagating theoretical and observational uncertainties.

- **Massive neutrinos and light relics** imprint themselves through the expansion history, the scale-dependent suppression of growth due to free streaming, and the BAO phase shift from relativistic species. Accurate modeling requires the separation of the cold+baryon and neutrino components, incorporation of scale-dependent growth, and careful treatment of degeneracies with other parameters. Current DESI+CMB bounds are already approaching the minimum mass scale allowed by oscillation data, and upcoming surveys will decisively test the normal hierarchy.

Despite the differences in physical origin, both cases share common analysis challenges: disentangling subtle signatures from degeneracies with late-time cosmology, galaxy bias, and observational systematics, maintaining theoretical accuracy across the wide dynamic range probed by modern surveys, understanding the impact of assumed parameter priors, and consistently combining information from multiple tracers, redshifts, and statistics.

The next 5-10 years will bring an unprecedented volume of high-precision LSS data from DESI, Euclid, SPHEREx, the Roman Space Telescope, Vera Rubin LSST survey, CMB–S4, and other facilities. For PNG, this will mean a transition from upper limits to regime-changing sensitivity at $f_{\rm NL} \lesssim 1$. For neutrinos, the combination of BAO, full-shape, lensing, and the CMB will either deliver a robust detection at the normal-hierarchy mass scale or point to new physics in the neutrino sector or cosmology itself.

Reaching these goals will require close interplay between theory and observation: improving perturbative modeling, developing efficient but accurate emulators for forward modeling, and building pipelines that can jointly fit diverse datasets with rigorous control of uncertainties. The reward is profound—direct measurements of the physics of inflation and the absolute neutrino mass scale, constraints on hidden relativistic sectors, and the potential discovery of physics beyond the standard cosmological and particle-physics paradigms.

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